

## MAIN TOPICS FOR TMA4230 FUNCTIONAL ANALYSIS

- Basic definitions: normed spaces, innerproduct spaces, Banach spaces, Hilbert spaces, completion of normed spaces, Banach algebras, Schauder basis. Examples.
- Basic notions: Linear operators, linear functionals, norm of linear operators, boundedness, continuity, adjoints of linear operators, extensions of linear operators, closed operators. Dual space, second dual space, reflexivity. Examples.
- Baire's category theorem: complete metric spaces, dense, separable, nowhere dense, first and second category. Statement of Baire's Theorem. Basic idea of its proof. Consequences.
- Geometry of Hilbert spaces (orthogonal complement, orthogonal projections), orthonormal sequences, total orthonormal sequences, Bessel's inequality, Parseval's relation, Fourier series in general Hilbert space.
- Hahn-Banach: Formulation, idea of the proof in the real/complex case. Hyperplane separation theorem, Minkowski functional. Consequences.
- Banach-Steinhaus / uniform boundedness theorem. Formulation, idea of the proof. Consequences.
- Open mapping theorem, closed graph theorem. Applications.
- Strong, weak, weak-star convergence. Definitions and basic facts. Strong and uniform convergence of linear operators. Banach-Alaoglu, Riesz's lemma and compactness of sets in normed spaces.
- Compactness in metric spaces and Banach spaces.
- Compact operators: Basic facts and definitions, characterizations. trace class and Hilbert-Schmidt operators (Schatten-von Neumann classes).
- Spectrum and resolvent of elements in Banach algebras. Basic facts and notions. Examples. Spectral mapping theorem.