TMA4230 FUNCTIONAL ANALYSIS, WEEK 9

LAST WEEK: Last week we first looked at the Stone-Weierstrass Theorem and how one can use the Banach-Alaoglu Theorem to prove the existence of weak solutions to partial differential equations. We then started on spectral theory. I first reviewed spectral theory in the finite dimensional case and then introduced the notion of the resolvent and the spectrum of a linear operator on a complex normed space.

- The Stone-Weierstrass Theorem .
- An application of the Banach-Alaoglu Theorem to PDEs.
- Spectral theory in finite dimensions (Section 7.1).
- Some basic concepts in spectral theory (Section 7.2).

THIS WEEK: This week we shall continue with spectral theory. I will first show that when we are dealing with bounded linear operators on a complex Banach space, then the notion of the resolvent and the spectrum simplifies to a notion which makes sense for elements of complex algebras. We will then study the spectrum and resolvent for elements of complex algebras and later for elements of complex Banach algebras, and show that the spectrum of an element of a complex Banach algebra (and hence for a bounded linear operator on a complex Banach space) is compact.

- Section 7.3–7.7 of the book.
- Page 61–65 of the notes.

NEXT WEEK: Hopefully we will finish Chapter 7 next week. We will show that the spectrum of an element of a complex Banach algebra is nonempty and find a formular for the *spectral radius*. For this we need to study vector valued holomorphis functions which I will introduce.

- Section 7.3–7.7 of the book.
- Page 61–65 of the notes.

EXERCISES FOR NEXT WEEK: 7.3.4-6, 7.4.4, 7.5.1, 7.7.4 and 7.7.5.