TMA4230 FUNCTIONAL ANALYSIS, WEEK 8

LAST WEEK: Last week we finished the introduction to topology by proving Tychonoff's theorem and we then looked at the Banach-Alaoglu Theorem.

- Compactness and Tychonoff's theorem (page 39–43 of the notes).
- The Banach-Alaoglu Theorem (page 52–54 of the notes).

THIS WEEK: This week we shall first look at the Stone-Weierstrass Theorem. Then I will briefly explain how to use the Banach-Alaoglu Theorem to solve partial differential equations. After that we will begin on spectral theory. As I mentioned last week we will do things in another order than the book does and instead follow the lead of the notes, but in the end we will cover all of Chapter 7 and page 61–65 of the notes. I expect to cover Section 7.1 and Section 7.2 which cover some basic concepts in spectral theory and spectral theory in finite dimension, and perhaps also begin on Section 7.6 and page 61–65 of the notes which introduce spectral theory for Banach algebras.

- The Stone-Weierstrass Theorem (extra notes which you can get in class or by sending me an email).
- An application of the Banach-Alaoglu Theorem to PDEs (extra notes which you can get in class or download from the web page).
- Spectral theory in finite dimensions (Section 7.1).
- Some basic concepts in spectral theory (Section 7.2).
- Spectral theory for Banach algebras (Section 7.6 and page 61–65 of the notes).

NEXT WEEK: Next week we will continue looking at properties of the spectrum and the resolvent of a linear operator. We will among other things show that the spectrum of a bounded linear operator on a complex Banach space is non-empty, closed and bounded and find a formula for the *spectral radius*.

- Spectral properties of bounded linear operators (Section 7.3).
- Further properties of resolvent and spectrum (Section 7.4).
- Spectral theory for Banach algebras (Section 7.6 and page 61–65 of the notes).

EXERCISES FOR NEXT WEEK: 7.1.10 ,7.1.15, 7.2.3 and 7.2.6 plus the following exercise.

EXERCISE: Show that the set of piecewise linear continuous functions on an interval $[a,b]$ is uniformly dense in $C[a,b]$.

Hint: Use Lemma 2 from the notes about the Stone-Weierstrass Theorem.