TMA4230 Functional analysis, Week 5

Last week: Last week we looked at adjoint operators between normed spaces and saw how this is related to the adjoint of a bounded operator between Hilbert spaces (Section 4.5). We then looked at reflexive spaces and showed that every Hilbert space is reflexive (Section 4.6). Finally we looked at Baire's category theorem and the uniform boundedness theorem (Section 4.7).

- Adjoint operators (Section 4.5).
- Reflexive spaces (Section 4.6).
- The Baire's category theorem and the uniform boundedness theorem (Section 4.7).

This week: This week we will look at strong and weak convergence (Section 4.8), convergence of sequences of operators and functionals (Section 4.9), and, if time permits, begin on the open mapping theorem (Section 4.12).

- Strong and weak convergence (Section 4.8).
- Convergence of sequences of operators and functionals (Section 4.9).
- The open mapping theorem (Section 4.12).

Next week: Next week we will first finish Section 4.12 about the open mapping theorem and then look at closed linear operators and the closed graph theorem (section 4.13). Then I will give a short introduction to topology with applications to functional analysis in mind.

- The open mapping theorem (Section 4.12).
- Closed linear operators and the closed graph theorem (Section 4.13).
- Page 32-43 of the notes.

Exercises for next week: 4.7.6, 4.8.1, 4.9.3 and 4.9.6 plus the following exercise.

Exercise: Show that there exists an $f \in C[0,1]$ which is not differentiable at any point in $[0,1]$.

Hint: For each $n \in \mathbb{N}$ let $A_n = \{f \in C[0,1] \mid \exists x \in [0,1] \forall y \in [0,1] : |f(x) - f(y)| \leq n|x-y|\}$. Show that each A_n is closed in $(C[0,1], ||\cdot||_\infty)$ and has empty interior and conclude that $C[0,1] \neq$

$\bigcup_{n\in\mathbb{N}} A_n$. Finally show that if $f \in C[0,1]$ is differentiable at a point $x \in [0,1]$, then f belongs to A_n for some $n \in \mathbb{N}$.