TMA4230 FUNCTIONAL ANALYSIS, WEEK 13

LAST WEEK: Last week we covered Section 9.4 about square roots of positive operators, Section 9.5–9.6 which deals with projection operators, Section 9.7 which is about spectral families and parts of Section 9.8 which deals with the spectral family of a bounded self-adjoint linear operator

- Square roots of positive operators (Section 9.4).
- Projection operators (Section 9.5–9.6).
- Spectral families (Section 9.7).
- The spectral family of a bounded self-adjoint linear operator (Section 9.8).

THIS WEEK: This week Takeshi Katsura is visiting the department of mathematical sciences. Katsura is one of the world's leading experts in operator algebra (a subfield of functional analysis) and an excellent lecturer. I have therefore arranged with Katsura that he will give a guest lecture Friday (at the usual time and the usual place) about semiprojectivity and graph C*-algebras. This will be a joint arrangment with the functional analysis operator algebra seminar. I will Thursday give an introduction to C*-algebras in general and graph C*-algebras in particular in order to prepare you for Katsura's talk.

NEXT WEEK: Next week (which is the last week of the semester) we will continue with the spectral theorem for self-adjoint operators. I hope to finish Section 9.8 and cover Section 9.9, and then briefly talk about Section 9.9 and 9.10

- The spectral family of a bounded self-adjoint linear operator (Section 9.8).
- The spectral theorem for bounded self-adjoint linear operators (Section 9.9).
- Extension of the spectral theorem to continuous functions (Section 9.10).
- Properties of the spectral family of a bounded self-adjoint linear operator (Section 9.11).

EXERCISES FOR NEXT WEEK: 9.8.1–9.8.4 plus the following two exercises.

EXERCISE 1: Let X be a locally compact Hausdorff space and suppose the C^*-algebra $C_0(X)$ is generated by a sequence of projections $\{p_n \mid n \in \mathbb{N}\}$ (i.e., $C_0(X) = C^*(\{p_n \mid n \in \mathbb{N}\})$). Show that the infinite sum $\sum_{n \in \mathbb{N}} p_n/3^n$ converges in norm to an element $x \in C_0(X)$ and that $C_0(X)$ is generated by this element.

EXERCISE 2: Let $E = (E^0, E^1, r, s)$ be a directed graph such that E^0 is finite. Show that $\sum_{v \in E^0} p_v$ is a unit for $C^*(E)$.