4.7.6) Let X be complete metric, and M a meager subset, i.e. $M = \bigcup_{n=1}^{\infty} A_n$, where the A_n's are nowhere dense.
If M^c also was meager, i.e. $M^c = \bigcup_{n=1}^{\infty} B_n$, for nowhere dense sets B_n, then $X = M \cup M^c$ would be a countable union of nowhere dense sets, which contradicts Baire's theorem.

4.8.1) Note first that f_t given by $f_t(x) = x(t)$ is a bounded linear (multiplicative) functional on $C[a,b]$ for each $t \in [a,b]$ (check!).
So if $(x_n) \subset C[a,b]$ and $x_n \xrightarrow{w} x \in C[a,b]$, then $f(x_n) \to f(x)$ for all f in $C[a,b]'$, in particular $f_t(x_n) = x_n(t) \to x(t) = f_t(x)$ for all $t \in [a,b]$

4.9.3) Suppose $T_n \to T$ in SOT, i.e. $\|T_n x - Tx\| \to 0$ for all $x \in X$
Pick $f \in Y'$, then $|f(T_n x) - f(Tx)| = |f(T_n x - Tx)| \leq \|f\| \, \|T_n x - Tx\| \to 0$
Hence $T_n \to T$ in WOT also (since limits are unique).

4.9.6) Suppose $T_n \to T$, then for all $\varepsilon > 0$, there exists N s.t.
$\|T_n x - Tx\| \leq \sup_{\|x\|=1} \|T_n x - Tx\| = \|T_n - T\| < \varepsilon$ for all $n > N$ for all $x \in S(X)$
Suppose that for all $\varepsilon > 0$, there exists N s.t. $\|T_n x - Tx\| < \varepsilon$ for all $n > N$, and all $x \in S(X)$. Then $\|T_n - T\| = \sup_{\|x\|=1} \|T_n x - Tx\| \leq \varepsilon < 2\varepsilon$, so $T_n \to T$.

Extra)

First we show that each A_n is closed. Suppose not, then there exists a sequence (f_i) in A_n, s.t. $f_i \to f$ in $C[0,1]$, but $f \notin A_n$.

Hence for each $x \in [0,1]$, there exists a $y \in [0,1]$ s.t. $|f(x) - f(y)| > n|x-y|$.

Let (x_i) be a sequence of elements s.t. $|f_i(x_i) - f_i(y)| \leq n|x_i - y|$ for all $i \in \mathbb{N}$ and $y \in [0,1]$. Fix $\varepsilon > 0$, then there exists j s.t. $\sup_{x \in [0,1]} |f(x) - f_i(x)| < \varepsilon$ for all $i > j$. Thus

$$|f_i(x_i) - f_i(y)| > |f(x_i) - f(y)| - 2\varepsilon > n|x_i - y| - 2\varepsilon \quad \text{for all } y \in [0,1],\ i > j$$

Since ε was arbitrary, this contradicts the fact that $f_i \in A_n$ for all i.

Next, A_n has empty interior. Fix $n \in \mathbb{N}$, and $f \in A_n$, then for each $\varepsilon > 0$ there exists a function $g \in C[0,1]$ s.t. $\|g\| < \varepsilon$ and $|g(x) - g(y)| > 3n|x-y|$ for x, y sufficiently close, e.g.

is the graph of g, with slope $4n$ of the straight lines.

Then $\|f+g\| < \|f\| + \varepsilon$, but $f+g$ is not in A_n, since there for all $x \in [0,1]$ exists $y \in [0,1]$ s.t. $|(f+g)(x) - (f+g)(y)| = |f(x) - f(y) + g(x) - g(y)| \geq |g(x) - g(y)| - |f(x) - f(y)| > 3n|x-y| - n|x-y| > n|x-y|$.

That is, for all $n \in \mathbb{N}$, $f \in A_n$ and $\varepsilon > 0$ there exists a function h $(= f+g)$ s.t. $h \in B(f, \varepsilon)$, the open ε-ball around f, but $h \notin A_n$.

If $f \in C[0,1]$ is differentiable at $x \in [0,1]$, then for each $\varepsilon > 0$ there exists $\delta > 0$ s.t. $|f(x) - f(y)| < (|f'(x)| + \varepsilon)|x-y|$ whenever $|x-y| < \delta$

For $y \notin B(x, \delta)$, $\sup_{y \notin B(x,\delta)} \frac{|f(x) - f(y)|}{|x-y|} \leq \sup_y \frac{1}{\delta}|f(x) - f(y)| = K_\varepsilon < \infty$ as f is continuous, so if N is an integer bigger than $\max\{K_\varepsilon, |f'(x)| + \varepsilon\}$ then $f \in A_N$

Since $C[0,1]$ is a complete metric space, Baire's theorem applies, and $C[0,1] \neq \bigcup A_n \supseteq \{f \mid f \text{ differentiable at some point}\}$.