THIRD WEEK HOMEWORK ASSIGNMENT

Problem 1. Prove that the elementary measure is sub-additive.

In other words, show that the map $m\colon \mathcal{E}(\mathbb{R}^d) \to \mathbb{R}$ satisfies

$$m(E \cup F) \leq m(E) + m(F)$$

for all elementary sets E and F.

Problem 2. Prove that the Cantor set is Jordan measurable and that its Jordan measure is 0.

Problem 3. Let $E \subset \mathbb{R}^d$ be any bounded set. Prove the following:

(a) $m^{*,J}(\overline{E}) = m^{*,J}(E)$, where $\overline{E}$ denotes the closure of E.

(b) $m_{*,J}(\mathring{E}) = m_{*,J}(E)$, where $\mathring{E}$ denotes the interior of E.

(c) E is Jordan measurable if and only if $m^{*,J}(\partial E) = 0$, where ∂E denotes the boundary of E.

Problem 4. Let $E := \mathbb{Q} \cap [0, 1]$. Prove the following

(a) $m_{*,J}(E) = 0$.

(b) $m^{*,J}(E) = 1$.

Conclude that E is not Jordan measurable.

More problems to be added later ...