LF err

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2 a) Lifting:

First we must construct a lifting of the boundary condition.

Let $R(x) = x$.

then $u = u_0 + R$, where u_0 satisfies homogeneous problem.

we then find

$$A(u_0 + R) = 0$$

hence

$$A u_0 = -AR = -\begin{pmatrix} h_0 + h_1 \\ \vdots \\ h_{M-2} + h_{M-1} \end{pmatrix}$$

(after bcs implemented)

and the matrix A is shown to be of the finite difference form, ie.

$$\begin{cases} 0.05\left(\dfrac{\mathring{u}_n - \mathring{u}_{n-1}}{h_{n-1}} + \dfrac{\mathring{u}_n - \mathring{u}_{n+1}}{h_n}\right) + \dfrac{2}{2}\left(\mathring{u}_{n+1} - \mathring{u}_{n-1}\right) = -h_n - h_{n-1} \\ \mathring{u}_0 = 0, \quad \mathring{u}_M = 0 \end{cases}$$

substituting $\mathring{u}_n = u_n - (h_0 + \dots + h_{n-1})$

$$\Rightarrow \quad \mathring{u}_n - \mathring{u}_{n-1} = u_n - u_{n-1} - h_{n-1}$$

and $\mathring{u}_{n+1} - \mathring{u}_{n-1} = u_{n+1} - u_{n-1} - h_{n-1} - h_n$

ie, we find the recurrence

$$\begin{cases} 0.05\left(\dfrac{u_n - u_{n-1}}{h_{n-1}} + \dfrac{u_n - u_{n+1}}{h_n}\right) + u_{n+1} - u_{n-1} = 0 \\ u_0 = 0, \quad u_M = 1 \end{cases}$$