TMA4215 Numerical Mathematics Autumn 2017

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Exercise set 2

1 Given

$$G(x) = \begin{pmatrix} \frac{1}{3}\cos(x_1 x_2) + \frac{1}{6} \\ \frac{1}{9}\sqrt{x_1^2 + \sin x_3 + 1.06} - 0.1 \\ -\frac{1}{20}\mathrm{e}^{-x_1 x_2} - \frac{10\pi - 3}{60} \end{pmatrix}$$

Show that the fixed point iterations $x^{(k+1)} = G(x^{(k)})$ converge towards a unique fixed point for all starting values $x^{(0)}$ in $D = \{x \in \mathbb{R}^3 : -1 \leq x_i \leq 1,\ i = 1,2,3\}$.

Verify the result numerically.

2 Consider the system of equations from Exercise 2,

$$x_1^2 + x_2^2 = 1,$$
$$x_1^3 - x_2 = 0.$$

This has two solutions, one in the region $-1 \leq x_1, x_2 \leq 0$ and one in $0 \leq x_1, x_2 \leq 1$. It is possible to show numerically that the iteration scheme based on the formulation

$$x_1 = \sqrt[3]{x_2},$$
$$x_2 = \sqrt{1 - x_1^2}$$

converges with appropriate starting values.

Explain why. How would you select starting values?

Hint: It is simpler to analyse results if you consider two subsequent iterations as one.

3 Given the linear system of equations

$$\begin{aligned} x_1 - 5x_2 + x_3 &= 7, \\ 10x_1 \quad + 20x_3 &= 6, \\ 5x_1 \quad - x_3 &= 4. \end{aligned}$$

Solve the equation by

a) Naive Gauss-elimination.
b) Gauss-elimination with partial pivoting.

Write down the LU factorization in both cases.

No solution will be given, but you can easily check your answers yourself. To see if you have done the pivoting right, check your result with MATLAB or Python.