Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4212
Numerical solution of differential equations by difference methods
Spring 2021

Exercise set 2

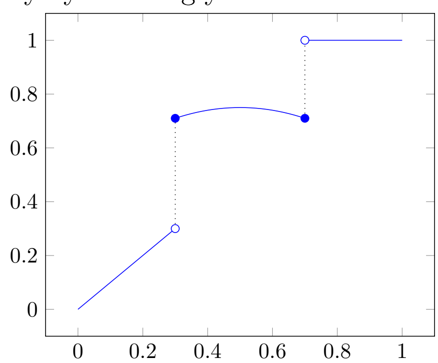
$\boxed{1}$ Consider the following heat equation on $x \in [0,1]$,

$$u_t = u_{xx}, \quad u(0,t) = u(1,t) = 0, \quad u(0,x) = \begin{cases} 2x, & 0 \le x \le 1/2, \\ 2-2x, & 1/2 \le x \le 1, \end{cases}$$

for $t > 0$ as we considered in Section 4.2.

a) Implement the Crank-Nicolsons method on your computer. Since this is a voluntary exercise, use any language you like. If you choose to use Matlab, the code is given in Section 4.2.4. Observe time evolution of the solution. When time t is sufficiently large, how the solution look like?

b) The heat equation is known to have smoothing property: even if the initial condition is discontinuous (but in L_2), the solution at any $t > 0$ is in C^∞. Check this property numerically by choosing your own discontinuous initial condition.



c) Modify your code for a general θ method, and compare the numerical solution (your choice of $\theta \neq 1/2$) with the Crank-Nicolsons method. Draw the convergence plot (in terms of M) of this method using as if the Crank-Nicolsons method is the analytic solution, where both methods should use the same number of points M. Theoretically, what convergence rate do you expect? And do you numerically see it?

d) Consider a modified problem

$$u_t = -u_{xx},$$

with the same boundary/initial conditions. This is known to be an ill-posed problem where the solution diverges analytically. Consider the θ method to numerically solve this problem. Prove that you cannot obtain $F-$stability with any choice of parameters (Hint: look at Section 5.5).

$\boxed{2}$ Solve the problems 3, 4 and 5 of the exercise 1 from 2020.