

# Elliptic equations

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Consider the Laplace equation,

$$\Delta u = u_{xx} + u_{yy} = 0$$

on the square  $(0, 1)^2$ . Use Dirichlet boundary conditions  $u = g(x, y)$  on  $\partial\Omega$ , where

$$\begin{aligned} g(0, y) &= 0, & 0 \leq y \leq 1, \\ g(x, 0) &= 0, & 0 \leq x \leq 1, \\ g(1, y) &= 0, & 0 \leq y \leq 1, \\ g(x, 1) &= \sin(\pi x), & 0 \leq x \leq 1. \end{aligned}$$

Verify that

$$u_e(x, y) = \frac{1}{\sinh(\pi)} \sin(\pi x) \sinh(\pi y)$$

is the exact solution to this problem.

**Task 1** Implement the five point formula for this problem. Use constant step-sizes  $h$  and  $k$  in each space direction (but allow for  $h \neq k$ ). Verify order 2 convergence in both space directions. *Hand-in: Plot of the numerical solution for  $h = k = 0.1$  and convergence plots that verify the correct order in both space directions.*