Hyperbolic equations

January 4, 2015

Exercise 1

Consider the advection diffusion problem

$$u_t + u_x = \nu u_{xx} + f, \quad t \geq 0, \qquad -1 < x < 1, \qquad f \equiv 1$$

with initial condition

$$u(x, 0) = \cos(\pi/2x)$$

and boundary conditions

$$u(-1, t) = 0, \qquad t \geq 0, \qquad u(1, t) = 0, \qquad t \geq 0.$$

Implement a finite difference discretization for this problem. Use different values of ν, say $\nu = 0.1$, $\nu = 0.01$ $\nu = 0.001$. What do you observe?

Exercise 2

Consider the problem

$$u_t + a u_x = 0, \qquad t \geq 0, \qquad -\infty < x < \infty$$

with initial condition

$$u(x, 0) = g(x).$$

Task 1 Implement the explicit difference formulae of chapter 7.3 in the note. In particular (7.11) and (7.12) verify that (7.12) is always unstable while (7.11) converges.

Task 2 Implement Lax-Wendroff and Leap-Frog and verify numerically their order in space and time.

Task 3 Consider finally the inviscid Burgers equation

$$u_t + \left(\frac{1}{2}u^2\right)_x = 0$$

discretize the time derivative and the space derivative in this equation using the same approximation as in formula (7.11). Verify numerically that the method converges.