Norwegian University of Science
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Department of Mathematical
Sciences

TMA4205 Numerical Linear Algebra Fall 2017

Exercise set 5

1 Compute the (reduced) singular value decomposition and the pseudoinverse of the matrix

$$A = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}.$$

2 Assume that $A \in \mathbb{R}^{n \times n}$ is skew-symmetric. Show that the singular values of A are precisely the absolute values of the eigenvalues of A.

3 Compute the (reduced) singular value decomposition of the matrix

$$A = \begin{pmatrix} 10 & 10 \\ -1 & 7 \\ 5 & 5 \\ -2 & 14 \end{pmatrix}.$$

Additionally, compute the pseudoinverse $A^\dagger$ of A and use it in order to solve the least squares problem

$$\min_{x \in \mathbb{R}^2} \frac{1}{2} \|Ax - b\|_2^2 \qquad \text{where } b = \begin{pmatrix} 7 \\ -5 \\ 1 \\ 1 \end{pmatrix}$$

4 Compute the pseudoinverse of the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}.$$

Using this particular matrix, show that the pseudoinverse of a matrix does not necessarily satisfy the relation $(A^\dagger)^2 = (A^2)^\dagger$.