

Example Exam Question
TMA4192 Differential Topology
NTNU, Spring 2023

All manifolds are smooth without boundary unless stated otherwise.

1. (a) Which of the following are global diffeomorphisms? Give reasons for your answers.

i. $g: (-\pi/2, \pi/2) \rightarrow \mathbb{R}, t \mapsto \tan t$

ii. $f: \mathbb{R} \rightarrow S^1 \subset \mathbb{R}^2, t \mapsto (\cos t, \sin t)$

- (b) State the Inverse Function Theorem.

Consider the map f in Part (a). Let $t_0 \in \mathbb{R}$ such that $\cos(t_0) > 0$. Let

$$V = \{(x, y) \in S^1 \mid x > 0\}$$

and consider the parameterisation of $S^1 \subset \mathbb{R}^2$ around $f(t_0) \in V$ given by

$$\psi: W = (-1, 1) \rightarrow V, \quad y \mapsto (\sqrt{1 - y^2}, y).$$

Then $d\psi_y: \mathbb{R} \rightarrow \mathbb{R}^2$ is given by

$$z \mapsto \left(\frac{-y}{\sqrt{1 - y^2}}, 1 \right) \cdot z.$$

- (c) Compute the derivative df_{t_0} explicitly and use this to conclude that the map f in Part (a) is a local diffeomorphism.

(21 points)