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Department of Mathematical Sciences

Examination paper for: TMA4190 Introduction to Topology

Examination date: May 27, 2020

Examination time (from-to): 09:00 - 13:00

Permitted examination support material: All support material is allowed

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OTHER INFORMATION
If a question is unclear/vague – make your own assumptions and specify in your answer the premises you have made. Only contact academic contact in case of errors or insufficiencies in the question set.

Saving: Answers written in Inspera Assessment are automatically saved every 15 seconds. If you are working in another program remember to save your answer regularly.

Cheating/Plagiarism: The exam is an individual, independent work. Examination aids are permitted. All submitted answers will be subject to plagiarism control. *Read more about cheating and plagiarism here.*

Citations: Be specific when citing a result from a book, paper or the lecture notes. You must include the author(s), the title of the work and, e.g., the theorem number if citing a specific theorem (or similar).

Example: By [Munkres, *Topology*, Theorem 59.3], the n-sphere S^n is simply connected for all integers $n \geq 2$
.

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1 # Problem 1

Let $X = \{a, b, c, d\}$.

Which of the following collections of subsets of X is **not** a topology on X?

Select one alternative:

- $\mathcal{T} = \{\emptyset, \{b, c, d\}, \{d\}, X\}$
- $\mathcal{T} = \{\emptyset, \{a\}, \{a, b\}, \{a, b, d\}, \{a, d\}, \{d\}, X\}$
- $\mathcal{T} = \{\emptyset, \{a\}, \{a, b, c\}, \{a, d\}, \{b, c, d\}, X\}$
- $\mathcal{T} = \{\emptyset, \{a\}, \{a, b\}, \{a, c, d\}, X\}$

2 # Problem 2

Let $X = \{a, b, c, d, e\}$ and let $\mathcal{T} = \{\emptyset, \{a\}, \{a, b, c\}, \{b, c\}, X\}$ be a topology on X.

Which of the following statements is **false**?

Select one alternative:

- The closure of $\{e\}, \overline{\{e\}},$ is equal to $\{d, e\}$.
- The interior of $\{d, e\}, Int(\{d, e\}),$ is equal to the empty set $\emptyset$.
- X is Hausdorff.
- X is compact.

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Problem 3

Let X be a topological space, and let $\mathcal{B}$ be a basis for the topology on X. Furthermore, let A be a subset of X
.

Show that $x \in \overline{A}$ if and only if $B \cap A \neq \emptyset$ for every basis element $B \in \mathcal{B}$ where $x \in B$.

(Here $\overline{A}$ denotes the closure of A.)

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Problem 4

Let $\mathbb{R}$ be the set of real numbers equipped with the standard topology, and consider the set of rational numbers $\mathbb{Q}$ as a subspace of $\mathbb{R}$.

Show that the subset $A = \{x \in \mathbb{Q} \mid -\sqrt{5} < x < 5\}$ of $\mathbb{Q}$ is both open and closed in $\mathbb{Q}$.

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Problem 5

Let $X = \{a, b, c, d\}$ be given the topology $\mathcal{T}_X = \{\emptyset, \{a\}, \{a, c, d\}, \{c, d\}, X\}$, and let $Y = \{1, 2, 3\}$ be given the topology $\mathcal{T}_Y = \{\emptyset, \{1\}, \{1, 3\}, Y\}$.

Find a basis for the product topology on $X \times Y$ expressed using bases for the topologies on X and Y, respectively.

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Problem 6

Let X be a topological space, and consider $I = [0, 1]$ as a subspace of $\mathbb{R}$ where $\mathbb{R}$ is given the standard topology. Furthermore, let the *cone on* X be the quotient space $CX = X \times I/{\sim}$, where $\sim$ is the equivalence relation on the product space $X \times I$ given by $(x, 0) \sim (x', 0)$ for all $x, x' \in X$.

Show that CX is path connected.

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Problem 7

Let X be a topological space, and let $A_1, A_2, \ldots, A_n$ be subspaces of X each of which is compact in X.

Show that $\bigcup_{i=1}^{n} A_i$ is compact in X.

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Problem 8

Let X be contractible space.

Show that X is path connected.

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Problem 9

Let B be a Hausdorff space, and let E be a topological space.

Show that if $p\colon E \to B$ is a covering map, then E must be Hausdorff.

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Problem 10

Let n be an integer that is greater than or equal to 3, and let $D^n = \left\{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n \mid \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2} \leq 1\right\}$ be considered as a subspace of $\mathbb{R}^n$ where $\mathbb{R}^n$ is given the standard topology.

Show that the inclusion map $i\colon D^n \setminus \{0\} \to D^n$ induces an isomorphism of fundamental groups.
(Here 0 denotes the origin in $\mathbb{R}^n$.)

You may assume as a known fact that the m-sphere S^m is simply connected where S^m is considered as a subspace of $\mathbb{R}^{m+1}$ and m is an integer that is greater than or equal to 2.

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