# What you should know about in order to understand manifolds:

**TMA 4190: Manifolds , spring 2011**

- topological space, basis for its topology, various topological properties such as : Hausdorff, compact, connected, locally Euclidean. Continuous function, homeomorphism
- topological manifold, atlas and charts, transition functions (for overlapping charts)
- smooth manifolds. smooth (differentable, or $C^\infty$) atlas, differentiable structure (maximal atlas), smooth map, diffeomorphism. Orientation, orientable or non-orientable manifold.
- Lie groups ? Here you must first of all know what is the (algebraic) definition of a group. But he Lie group is also a smooth manifold, and now you should be able to explain the definition. Give also some examples.
- many examples of manifolds, compact or noncompact, their dimension etc.
- rank of a smooth map, immersion, imbedding, submersion, inverse function theorem
- regular points, regular values. Some version of Sard's theorem concerning regular values.
- submanifolds . (The imbedding of a manifold into another is a submanifold. But make sure you understand - with a counterexample- why an injective immersion may not be an imbedding)
- tangent space, tangent vectors. Various interpretations/definitions of a tangent vector, e.g. a local derivation.
- vector bundles , cross sections, product bundle, trivial bundle, isomorphism and homomorhism between two bundles.
- tangent bundle, cotangent bundle. Vector fields, covector fields (1-forms), expressions in local coordinates
- partition of unity, any application?
- imbedding in Euclidean space. (Whitney's imbedding theorem)

- Riemannian structure. Explain what is a Riemannian metric on a smooth manifold. The classical notation for the metric in local coordinates $(x_1, x_2, ..., x_n)$ is the square of the "arc-length" element $ds$, namely

$$ds^2 = \sum g_{ij} dx_i dx_j$$

  Make sure you understand what his means. Explain also the connection with the definition of the metric as a bilinear form on the tangent spaces, having certain properties.

- dynamical systems (1. order ODE). Explain the 1-1 correspondence between a dynamical system and a vector field

- flows (local or global) and one-parameter groups.

- the relationships between the concepts : vector field, dynamical system, flow, one-parameter group

- vector fields regarded as derivations , namely linear differential operators of order 1

$$X : C^\infty(M) \to C^\infty(M)$$

  In local coordinates $(x_1, x_2, ..., x_n)$, what is the expression for $X$ ?

- The Lie-bracket of vector fields , $[X, Y] = XY - YX$ (composition of operators).

- Lie algebra. Give examples and calculate the bracket to illustrate.