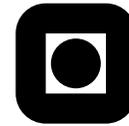Department contact under exam:
Andrew Stacey, phone (735) 90154

English version

Manifolds (TMA4190): Final Exam

Saturday 6th June 2009
Time: 9:00–13:00
Permitted Examination Aids: D
No written or handwritten examination support materials are permitted.
Calculator: Citizen SR-270X or Hewlett Packard HP30S

Problem 1.

Give a definition of a smooth manifold.

For each part of your definition, give a short explanation of what it means
and why it is included.   *(10 points)*

Problem 2.

Let $S^2 = \{(x, y, z) \subseteq \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$. Describe the standard smooth
manifold structure on S^2 and show that it really is a smooth structure on
S^2.   *(10 points)*

Problem 3.

The following is a list of properties that a smooth map between manifolds
can have.

   i. diffeomorphism
  ii. embedding
 iii. fibre bundle (i.e. the map in question is the projection map of a fibre
     bundle)
 iv. homeomorphism
  v. immersion
 vi. submersion

Give a list of which properties imply which other properties. Give ex-
amples of functions which show that the reverse implications are not
true.

As an example, a function which is a diffeomorphism is automatically a
local diffeomorphism so the property of being a diffeomorphism implies
that of being a local diffeomorphism. However, the reverse implication
is false as evidenced by the function $\mathbb{R} \to S^1$, $t \mapsto (\cos t, \sin t)$ which is a
local diffeomorphism but not a diffeomorphism.

(10 points)

Problem 4.

i. For each $a \in \mathbb{R}$, define a map $f_a \colon \mathbb{R} \to \mathbb{R}^2$ by

$$f_a(t) = (t^2, t(t^2 - a)).$$

For which values of a is this an immersion and for which values of a is this an embedding? *(6 points)*

ii. For $a \in \mathbb{R}$, define a map $g_a \colon \mathbb{R}^2 \to \mathbb{R}$ by

$$g_a(x, y) = y^2 - x^3 - ax.$$

Find the critical values of g_a. *(4 points)*