This problem set consists of three problems with a total of 10 subproblems. Each subproblem counts 10% towards your final grade. Show your work.

Problem 1

a) Find all 3-cyclotomic cosets for $n = 13$.

b) Find all cyclic codes of length 11 over $\mathbb{F}_3$. Determine their dimension and give a lower bound for their minimum distance.

c) Let $\mathcal{C}$ be a cyclic code and let $\mathcal{C}'$ be this code shortened on the last coordinate. Is $\mathcal{C}'$ cyclic?

Problem 2 In this problem we shall consider polynomials over $\mathbb{F}_2$.

a) Show that $f(X)$ is primitive if and only if $f^*(X)$ is primitive. (Note: $f^*(X)$ is the reciprocal polynomial of $f(X)$.)

b) Which of the irreducible polynomials $1 + X + X^2$, $1 + X + X^3$ and $1 + X^3 + X^6$ are primitive?

Problem 3

a) Explain what a catastrophic convolutional encoder is, and briefly explain why we do not want to use a catastrophic encoder.

b) Is $G = (\ 1 + X^4 + X^5,\ 1 + X + X^2 + X^5\ )$ catastrophic? If so, find a non-catastropic encoder for essentially the same convolutional code.

c) Determine the free distance of $G' = (\ 1 + X + X^2,\ 1 + X^2\ )$.

d) Explain how the Viterbi algorithm can be modified to deal with erasures.

e) Puncturing convolutional codes is done by removing a fixed pattern of of t symbols from every block of s codeword frames.

Suppose we remove t out of every sn symbols in an (n, k)-convolutional code. What is the resulting code rate?

Explain how to use the modified Viterbi algorithm from **d)** to decode punctured convolutional codes.