NTNU – Trondheim
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Department of Mathematical Sciences

Examination paper for **TMA4185 Coding theory**

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Examination date: June 11, 2016

Examination time (from–to): 09:00–13:00

Permitted examination support material: All printed and hand-written support material is allowed. A specific basic calculator is allowed.

Other information:

Language: English

Number of pages: 2

Number of pages enclosed: 3

Checked by:

Date Signature

Problem 1 In this problem we will be working with codes over $\mathbb{F}_3$.

a) Find a generator matrix for the smallest linear code $\mathcal{C} \subseteq \mathbb{F}_3^6$ that contains the code words $(1,1,2,1,0,1)$, $(2,2,1,1,1,1)$, $(1,1,1,0,0,2)$ and $(2,2,2,1,2,2)$. What is the dimension of the code?

b) Find a generator matrix on standard form for the code $\mathcal{C}$ if such a matrix exists. If such a matrix does not exist, find a generator matrix on standard form for a permutation-equivalent code.

Find a parity check matrix for $\mathcal{C}$.

c) What is the minimum distance for $\mathcal{C}$? Is the code perfect? Is it an MDS code?

d) Make a table with the syndomes of all errors with weight 1.

You have received $(1,0,1,2,1,1)$. Find the nearest code word in $\mathcal{C}$ using syndrome decoding.

e) Explain what a puncture of a code is. Show that a puncture of a linear code is also a linear code.

Let $\mathcal{C}^*$ be the puncture of $\mathcal{C}$ in the fourth coordinate. What is the dimension and minimum distance of $\mathcal{C}^*$.

Problem 2 Let $\mathbb{F}_8$ be $\mathbb{F}_2[Y]/\langle Y^3+Y^2+1\rangle$. Let $\alpha = Y + \langle Y^3+Y^2+1\rangle$. Let $\mathcal{C}$ be the cyclic code of length 7 over $\mathbb{F}_8$ defined by the zeros $\{1,2,3,4\}$.

a) What is the dimension of the code? How many errors can the code correct? How many erasures can the code correct?

Find a generator polynomial for the code $\mathcal{C}$.

b) You have received

$$\blacksquare X^6 + \alpha^2 X^5 + \alpha^2 X^4 + \left(\alpha^2+1\right) X^3 + X + \blacksquare$$

where $\blacksquare$ denotes erasures. If there are no other errors than the erasures, what code word was sent?

Problem 3 Does there exist a code of $\mathbb{F}_8$ with length $n = 35$, minimum distance $d \geq 7$ and at least 2^{28} code words? What is the size of the biggest code you know exists with $n = 35$ and $d \geq 7$?

Problem 4 In this problem, we shall work with convolutional codes over $\mathbb{F}_2$.

a) Explain why the generator matrix $G = (1 + D^2, 1 + D^2 + D^3 + D^4)$ is bad and find a better generator matrix. Use the generator matrix you suggest to encode the message 10 11 00.

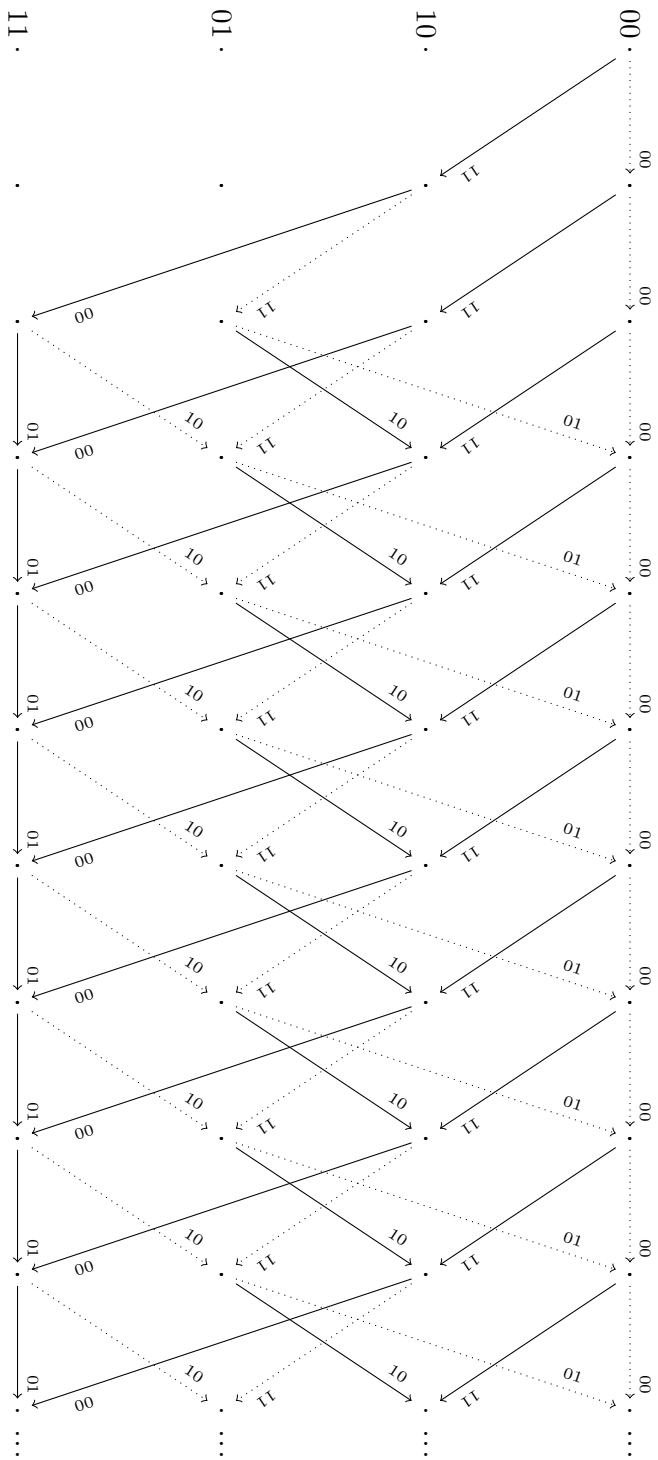
b) You have received

$$11\,11\,10\,01\,10\,10\,11\,01\,00.$$

The original message was encoded using the generator matrix $G = (1+D, 1+D+D^2)$. You know that the original message ended with two zeros. Use the Viterbi algorithm to decode the message.

Hint: You can use and hand in the attachment if you want to.

Trellis diagram for $G = (1 + D, 1 + D + D^2)$.



Trellis diagram for $G = (1 + D, 1 + D + D^2)$ (draft).