



Faglig kontakt under eksamen:
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Engelsk

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Tid: 9:00–14:00

Hjelpemidler: B-All printed and written aids permitted. Specific, approved calculator allowed.
Sensurdato: 4. juni 2004

Problem 1

Consider binary linear (n, k, d) codes.

- Suppose the word length is $n = 19$ and the distance is $d = 5$. What upper bound on the dimension k do we get from the Hamming bound?
- For which word lengths n does there exist a code with distance $d = 3$ and information rate $k/n = 1/2$?
- Does there exist a code with information rate $k/n \geq 2/3$, word length $n > 6$ and with $d/n = 1/2$?

Problem 2

The code is the Golay code C_{23} . Decode the received word $w = 100\ 101\ 011\ 000\ 111\ 000\ 100\ 00$.

Problem 3

Let C be the $RS(2^4, 5)$ code with generator polynomial $g(x) = (\beta+x)(\beta^2+x)(\beta^3+x)(\beta^4+x) = \beta^{10} + \beta^3x + \beta^6x^2 + \beta^{13}x^3 + x^4$, where β is a primitive element in the field $GF(2^4)$ constructed using the polynomial $1 + x + x^4$.

How many errors can C correct? Decode the received word $w = \beta^3\beta^{11}\beta^5001000000000$.

Problem 4

Let C be the binary cyclic linear code of length 15 with generator polynomial $g(x) = 1 + x + x^2 + x^3 + x^6$. This code is 3 cyclic burst error correcting. (You don't have to show this.) Suppose the word $w = 11000\ 10000\ 00000$ is received. Find an error pattern e with cyclic burst length at most 3 such that $w + e$ is a codeword.

Problem 5

Let C be a binary linear code. Let C' denote the subset of C consisting of all words in C with even weight.

- Show that C' is a linear code. Show that if C is cyclic, then C' is cyclic.
- Let $\dim C$ denote the dimension of C . Show that $\dim C' = \dim C$ or $\dim C' = \dim C - 1$.
- Suppose

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

is a generator matrix for C . Find a generator matrix for C' .

- Suppose C is the smallest cyclic linear code containing the word 010001100. Find the generator polynomial of C' .