Norges teknisk–
naturvitenskapelige universitet
Institutt for matematiske fag

Faglig kontakt under eksamen:
Navn: Dag Madsen
Telefon: 73 59 66 82/ 73 59 35 20

EKSAMEN I EMNE SIF5032, KODETEORI

Engelsk
Mandag 5. mai 2003
Tid: 9:00–14:00

Hjelpemidler:B-All printed and written aids permitted. Specific, approved calculator allowed.

Sensurdato: 26. mai 2002

Problem 1

Let C be the linear code with parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

a) List the cosets of C.

b) Construct a standard decoding array (using IMLD) for C, and decode the received words $w_1 = 011100$ and $w_2 = 111100$.

Problem 2

Consider binary linear (n, k, d) codes.

Suppose the length is $n = 12$ and the distance is $d = 4$. What upper bound on the dimension k do we get from the Hamming bound? What is the largest possible value for k for which the Gilbert-Varshamov bound says there exists a $(12, k, 4)$ code?

Problem 3

Let C be the smallest linear cyclic code containing the word 0101100.

a) Find the generator polynomial of C and a generator matrix for C.

b) Find the generator polynomial of the dual code $C^{\perp}$. Which of the codes C and $C^{\perp}$ has largest distance?

Problem 4

Let C be the $RS(2^3, 5)$ code with generator polynomial $g(x) = (\beta+x)(\beta^2+x)(\beta^3+x)(\beta^4+x) = \beta^3 + \beta x + x^2 + \beta^3 x^3 + x^4$, where β is a primitive element in the field $GF(2^3)$ constructed using the polynomial $1 + x + x^3$.

a) How many errors can C correct? Decode the received word $w = (1, 0, 1, 0, 0, 0, \beta)$.

b) Assume a word w is received with 2 erasures. How many errors (non-erasures) can C correct in addition to the two erasures? The word $w = (1, *, *, 1, 0, 0, 0)$ has one error except the erasures. Find the location of that error.

Problem 5

Let C_1 and C_2 be two binary linear cyclic codes of the same length. Show that their intersection $C_1 \cap C_2$ is also a linear cyclic code.

Suppose C_1 and C_2 both have length 21, and that C_1 has generator polynomial $g_1(x) = 1 + x^3$, while C_2 has generator polynomial $g_2(x) = 1 + x + x^2 + x^4$. What is the generator polynomial of $C_1 \cap C_2$?