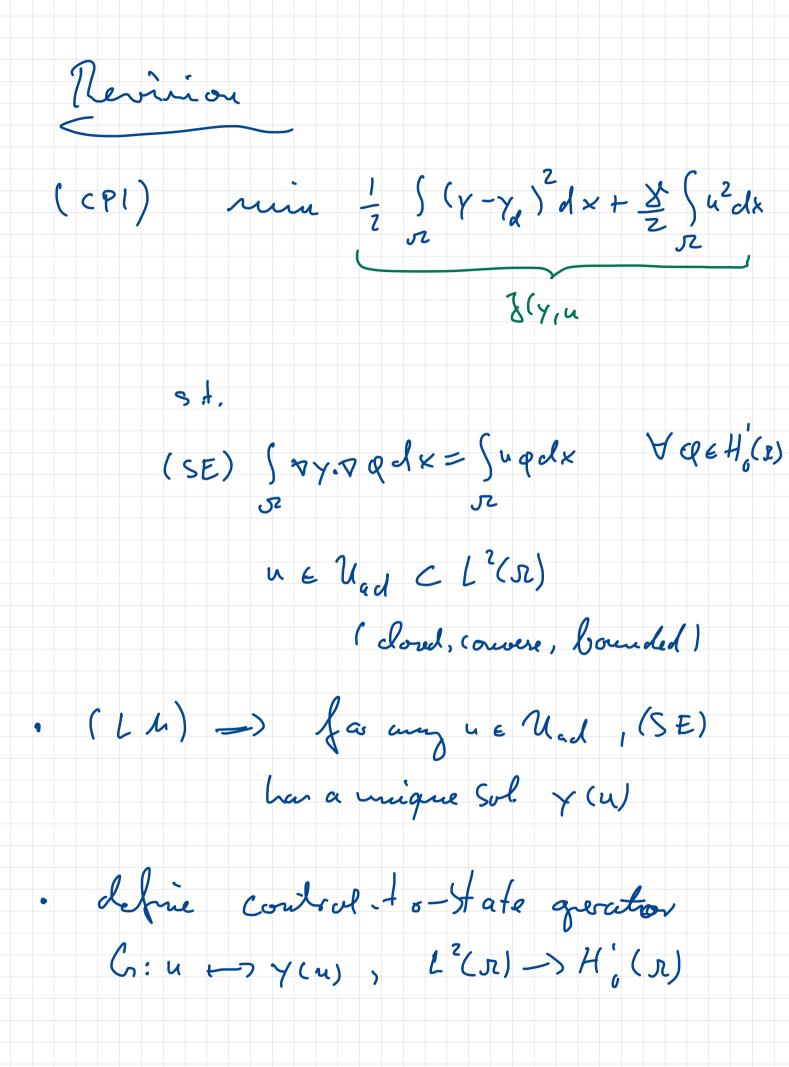
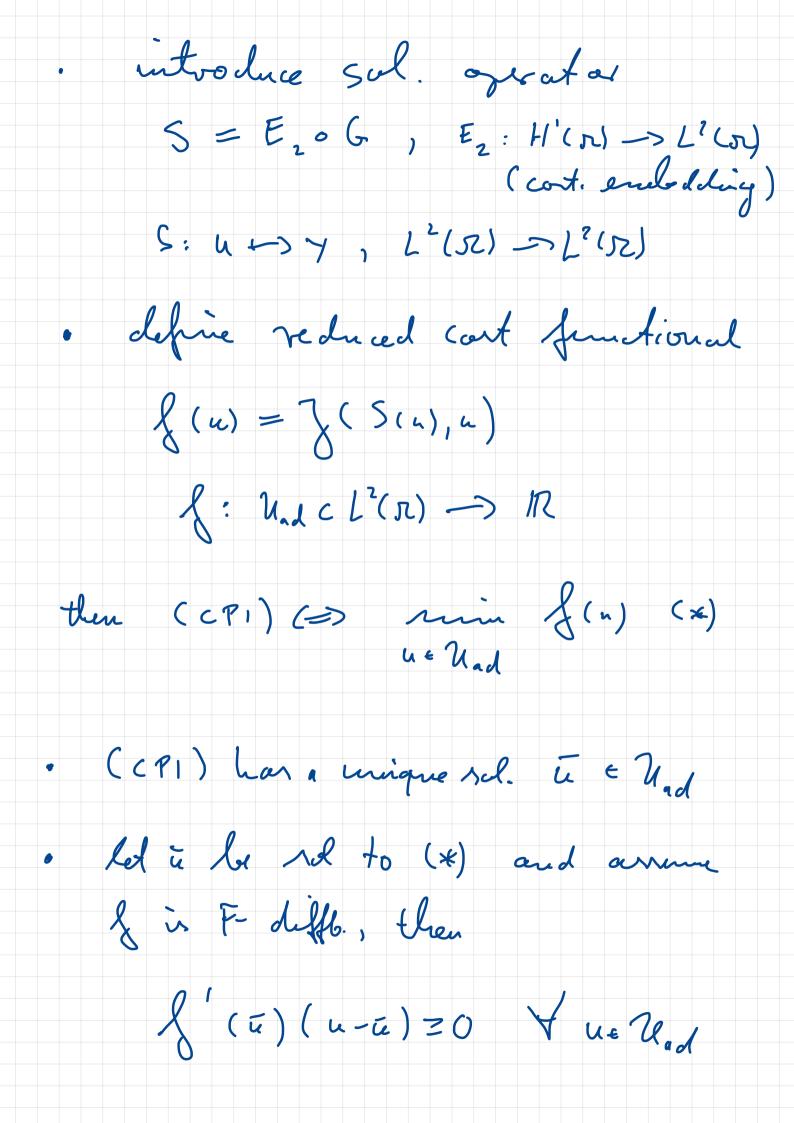
Optimization II

Lecture 9, Feb. 2

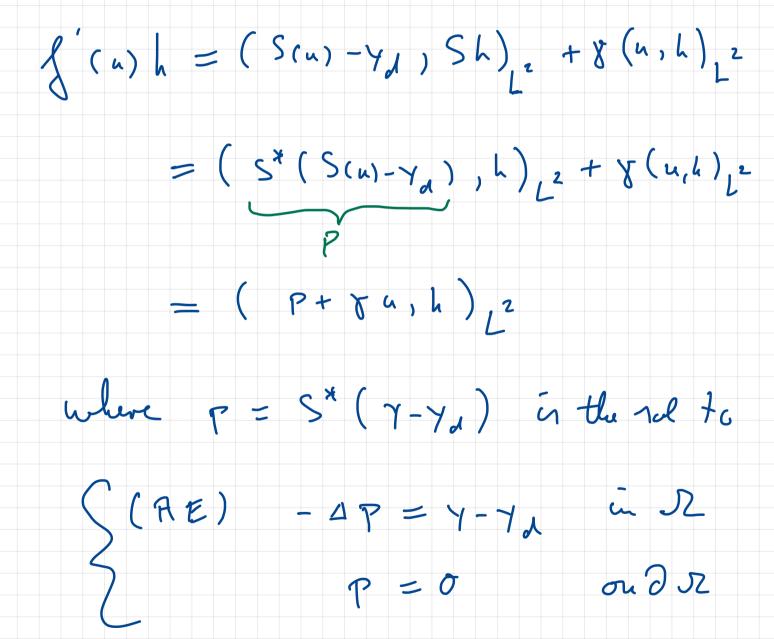


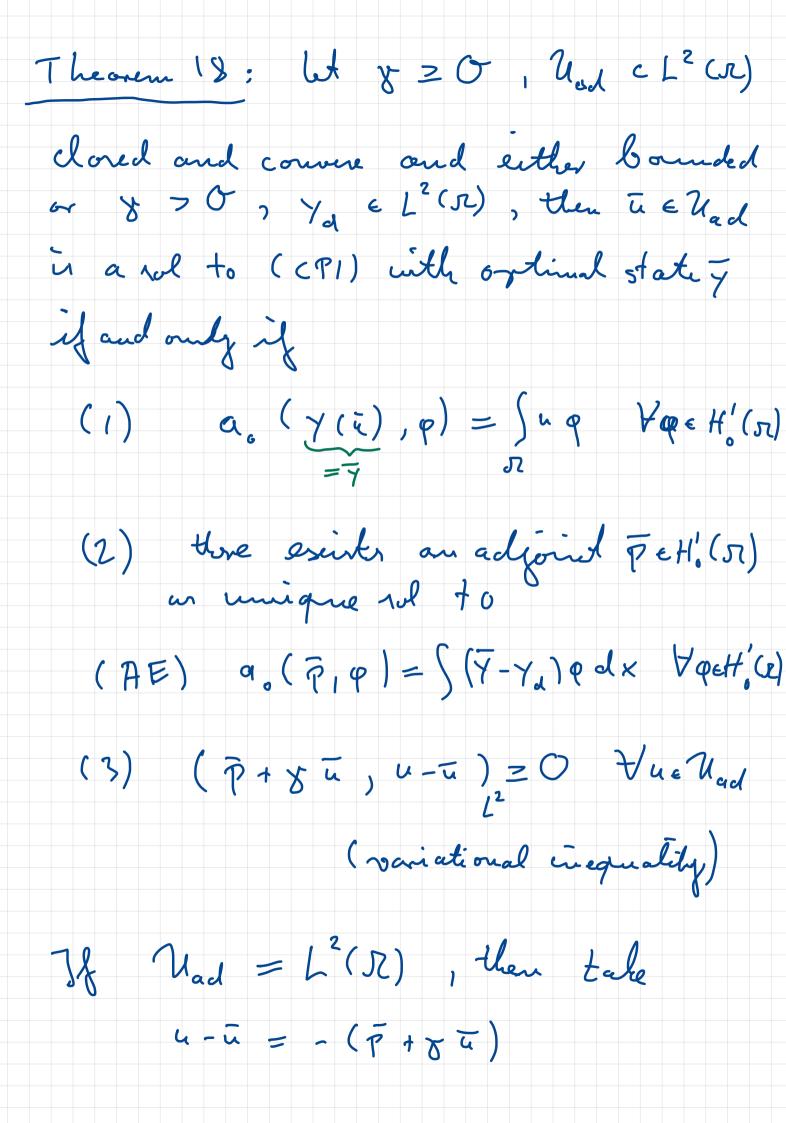


## where $\int : L^2(\mathcal{R}) \longrightarrow L(L^2(\mathcal{R}), \mathbb{R})$

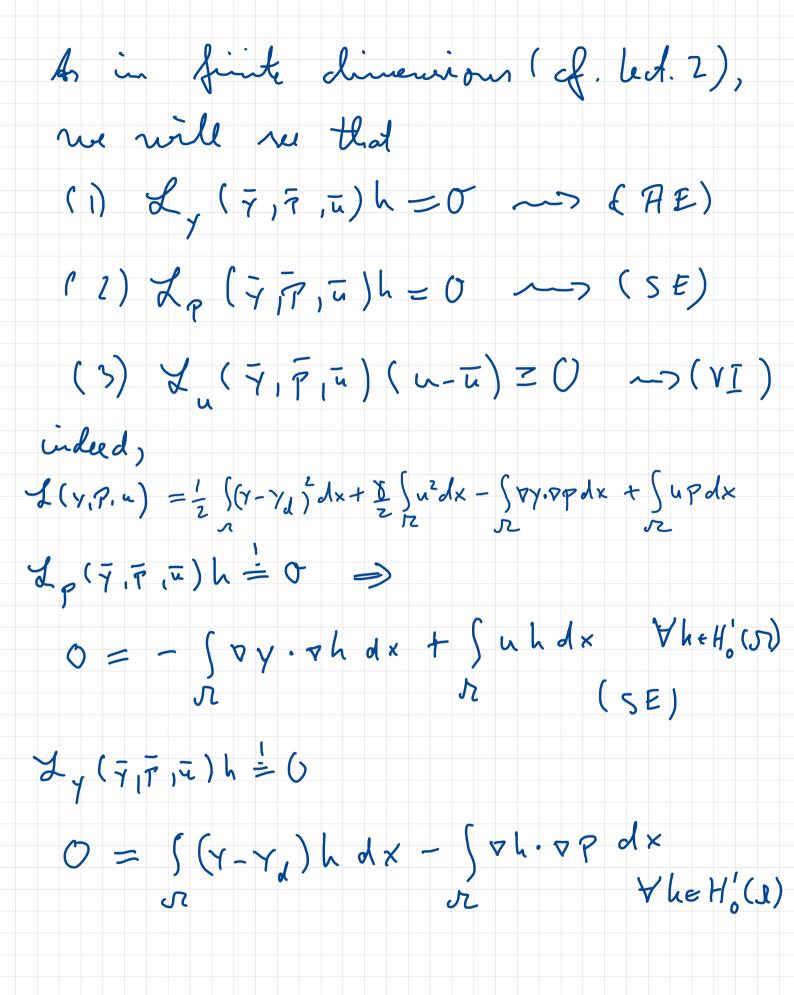
•  $\int a \left\{ \left( u \right) = \frac{1}{2} \int \left( \int \left( u \right) - Y_d \right)^2 dx + \prod_{z \in \mathcal{I}} \int u^2 dx$ 

 $= \frac{1}{2} \| S(u) - \gamma_{a} \|_{L^{2}}^{2} + \sum_{n=1}^{\infty} \| u \|_{L^{2}}^{2}$ 





 $= \sum - \| \overline{P} + \sqrt{n} \|^2 = 0$ =  $\overline{u} = -\frac{1}{8}\overline{P}$ 19: let Uad = 12 (52) and z = 0, Corollony in the rul to (CPI) ~> then u  $(\overline{\gamma},\overline{P}) \in H'_{o}(\overline{x}) \times H'_{o}(\overline{x})$  Noberes  $q_{o}(\overline{\gamma}, \varphi) = -\frac{1}{8} \int_{\Omega} \overline{P} \varphi dx \quad \forall \varphi \in H_{i}(\sigma)$  $\alpha_{(\overline{p},q)} = \int_{\Sigma} (\gamma - \gamma_{a}) q dx \quad \forall q \in H'_{a}(s)$ Amark: An eary way of deriving the optimality uptimes lay uning the lagrangian, i.e.,  $\mathcal{L}(Y, P, u) = \mathcal{J}(Y, u) - a_{o}(Y, P) + (u, P)$  $=\frac{1}{2}\int (Y-Y_d)^2 dx + \sum_{i=1}^{n} \int u^2 dx - \int \nabla Y \cdot \nabla p dx + \int u p dx$ 



 $\mathcal{J}(Y, P, u) = \frac{1}{2} \int (Y - Y_d)^2 dx + \sum_{i=1}^{n} \int u^2 dx - \int \nabla Y \cdot \nabla P dx + \int u P dx$ 

 $\mathcal{J}_{u}(\bar{\gamma},\bar{P},\bar{u})h = \mathcal{J}'(\bar{u})h$ 

 $= \chi \int \overline{u} h dx + \int h \overline{p} dx = \int (\chi \overline{u} + \overline{p}) h dx$   $T \qquad T \qquad T \qquad T$ 

Now, we try to further exploit (VI)

(YI)  $(\overline{P}+\overline{Y}\overline{u}, u-\overline{u})_{2^2} = 0$ 

 $(=) (\bar{\varphi} + \chi \bar{\mu}, \bar{\mu})_{L^{2}} \leq (\bar{\varphi} + \chi \bar{\mu}, \mu)_{L^{2}}$ 

hence  $(\overline{p} + \overline{y}\overline{u}, \overline{u}) = \min_{u \in \mathcal{U}_{ad}} (\overline{p} + \overline{y}\overline{u}, \overline{u})_{L^2}$ 

Next, ne introduce box contrainte:

 $\mathcal{N}_{ad} = \left\{ \left| \mathcal{L}_{ad}^{2}(\mathcal{R}) \right| \left| \left| \mathcal{L}_{ad}^{2}(\mathcal{R}) \right| \right| \left| \left| \mathcal{L}_{ad}^{2}(\mathcal{R}) \right| \right| \right\} = \left\{ \left| \mathcal{L}_{ad}^{2}(\mathcal{R}) \right| \left| \left| \mathcal{L}_{ad}^{2}(\mathcal{R}) \right| \right| \left| \left| \mathcal{L}_{ad}^{2}(\mathcal{R}) \right| \right| \right\}$ 

with 7, 9, EL®

then Uad · · loved, convex and bounded  $\dot{m} L^2(\Lambda)$ Now, we show, that (VI) can be dis curred point rerre a.e. in R: lemma 20. (VI) (=>  $(\overline{p}(x) + \overline{y}\overline{u}(x))(\underline{y} - \overline{u}(x)) \ge O$ ∀ ξ ∈ [ ℓ, (x), ξ, (x)] a. e. in JZ Proof: Define Z(x) = P(x) + Y u(x)) 2 is liber que meanvalle, her ce almont all X. E S are libergue pointe i.e., thre holds  $\lim_{S \to O} \frac{1}{13_{S}(r_{0})} \left\{ \frac{1}{2} (x) dx = \frac{1}{2} (x_{0}) \right\}$ Λ. (×.) 3

