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Optimization II

Problem sheet 1

1. Show that

$$\|f\|_2 = \Big(\int_a^b f^2 dx\Big)^{\frac{1}{2}}$$

defines a norm on $C[a, b]$.

2. Let $A \in I\!R^{n,n}$, $B \in I\!R^{n,m}$ and consider the finite-dimensional optimal control problem

$$\min \frac{1}{2}|y - y^d|^2 + \frac{\gamma}{2}|u|^2$$

subject to the state eequation

$$Ay = Bu$$

and the control constraint

$$u \in U_{ad} \subset I\!R^m$$

in the case of box constraints

$$U_{ad} = \{u \in I\!R^n | u^a \leq u \leq u^b\}$$

with $u^a \leq u^b \in I\!R^m$. Show that (LICQ) holds in this case and derive the KKT conditions for this problem.

3. Consider the boundary value problem

$$y'' + p(x)y' + q(x)y = f(x) \text{ in },\quad y(0) = y(1) = 0,$$

with $p \in C^1[0, 1]$, $q \in C[0, 1]$. Derive the weak formulation in the appropriate function space and show that it has a unique solution if $p' - 2q > 0$. Is this also true for $p' - 2q = 0$?

4. Show that

$$F : C[0,1] \to C[0,1], \quad u \mapsto \int_0^1 \Big(u(t)^2 e^{t-7} + (3u(t)+13)\sqrt{t} - 17\tan(\ln(1+t^2))\Big)\, dt$$

is Fréchet- differentiable and compute its derivative.

5. Consider the function $f : I\!R \to I\!R$, $x \mapsto x^3$. For which combinations of $p, q \in [1, \infty]$, is the Nemytzki-operator

$$f : L^p(\Omega) \to L^q(\Omega),\ u \mapsto f(u)$$

Fréchet- differentiable and why?

6. Show that $\sin : L^p(\Omega) \to L^q(\Omega)$ is F-differentiable if $p = q = \infty$ or $p > q \geq 1$.