TMA4183 Opt. II
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Exercise set 7

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Please read sections 4.1–4.2 in [Tr].

1 Let Assumptions 4.2 (i), (iii) and Assumptions 4.3 hold. Follow the ideas of Theorem 4.4 to show the existence and uniqueness of the solution to the semi-linear BPV for Laplace (or other uniformly elliptic operator as defined in (2.19)) operator with Dirichlet boundary conditions:

$$\begin{aligned} -\Delta y + d(x,y) &= f, \qquad && \text{in } \Omega \\ y &= 0, && \text{on } \Gamma. \end{aligned}$$

2 In several places in Section 4.2, for example in the proof of Theorem 4.5 or 4.7, the inequality

$$\|y\|_{L^\infty(\Gamma)} \leq \|y\|_{L^\infty(\Omega)}, \qquad \forall y \in H^1(\Omega) \cap L^\infty(\Omega),$$

is used. Prove it! (this is Exercise 4.1 in [Tr], see the hints there).

Additional hints:

a) Show that if $\|y_n - y\|_{H^1(\Omega)} \to 0$ then $|y_n| \rightharpoonup |y|$, weakly in $H^1(\Omega)$. (It is sufficient to show that $|y_n| \rightharpoonup |y|$, weakly in $L^2(\Omega)$ and $D_i|y_n| \rightharpoonup D_i|y|$, weakly in $L^2(\Omega)$.)

b) The previous point shows that if $\|y_n - y\|_{H^1(\Omega)} \to 0$ then $P_{[-c,c]}(y_n) \rightharpoonup P_{[-c,c]}(y)$, weakly in $H^1(\Omega)$.

c) The trace operator trace : $H^1(\Omega) \to L^2(\Gamma)$ is *compact* and therefore maps weakly convergent sequences into strongly convergent ones. Use this information to prove the final result. Recall that convergence in L^2 implies convergence almost everyehere, up to a subsequence.