Norwegian University of Science
and Technology
Department of Mathematical
Sciences

TMA4183 Opt. II
Spring 2016

Exercise set 4

Reading material: Section 2.8–2.10 from [Tröltsch].

1 Exercise 2.11 [Tr]:
Let real numbers u_a, u_b, β, p and some $\lambda > 0$ be given. Solve the quadratic optimization problem in $\mathbb{R}$,

$$\min_{v\in[u_a,u_b]} \left\{ \beta p v + \frac{\lambda}{2} v^2 \right\},$$

by deriving a projection formula of the type (2.58) on page 70.

2 Exercise 2.19 [Tr]:
Apply the formal Lagrange method to the problem with boundary control of Dirichlet type:

$$\min \int_\Omega |y - y_\Omega|^2 \,\mathrm{d}x + \lambda \int_\Gamma |u|^2 \,\mathrm{d}s =: J(y,u),$$

subject to

$$\begin{aligned} -\Delta y &= 0, \\ y|_\Gamma &= u, \\ -1 \le u(x) &\le 1. \end{aligned}$$

3 Exercise 2.15 [Tr]:
Derive the necessary optimality conditions for the linear optimal control problem on page 79:

$$\min\ J(y,u,v) := \int_\Omega (a_\Omega y + \lambda_\Omega v)\,\mathrm{d}x + \int_\Gamma (a_\Gamma y + \lambda_\Gamma u)\,\mathrm{d}s,$$

subject to

$$\begin{aligned} -\Delta y &= \beta_\Omega v, && \text{in } \Omega, \\ \partial_\nu y + \alpha y &= \beta_\Gamma u, && \text{on } \Gamma, \\ v_a(x) \le v(x) &\le v_b(x), && \text{a.e. in } \Omega, \\ u_a(x) \le u(x) &\le u_b(x), && \text{a.e. on } \Gamma, \end{aligned}$$