PROJECT DESCRIPTION

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Abstract. The project reports in Optimisation I are expected to be written in the form of scientific papers, and not merely as homework assignments. This short note is intended to provide some advice on how to write mathematical reports and scientific writing in general, with a special emphasis on texts concerned with applied or numerical mathematics.

1. Background material

This note is based on (and to a large degree identical to) a note for the 2017 edition of the class TMA4180, Optimisation I, as well as a note for the course TMA4215, Numerical Mathematics, in 2016 [Kvæ16]. It is intended to provide some ideas about scientific writing, particularly in the case of texts in applied or numerical mathematics.

2. On writing

Say something. The most important point when writing a text is to say something. In order to do that, you actually have to *have something to say*. This not only means that you should avoid writing about nothing, which would be hard within the constraints of this project anyway, but also that you should not try to say too many things at once. One of the difficulties you might face during this project is a strict limitation to at most ten pages, while you could easily produce numerical results covering fifteen. It will be much easier to select numerical examples, if you have already decided, what you want to say with them. Also keep in mind that you can include additional numerical examples in the jupyter notebook.[1]

Think about your audience. Before you start writing anything, think briefly about the audience you want to reach. It should make a difference, whether you write a text for specialists in numerical mathematics or first-year students—or somebody who has no mathematical background at all. In particular, this will be important if you have to decide how detailed you have to be, and what you can assume to be "common knowledge."

For this project, it may help to think that you are writing for fellow students in your study programme who have some background in optimisation (for instance, because they have attended the class in earlier years) but are not familiar with the problem posed in the project. There is thus no need for you to explain what a line search method or a Quasi-Newton method is, but you need to include a brief explanation of what the project is about. A general advice is also to try to write anything in such a way that you yourself are still able to understand it in a few years.[2]

Date: 27th February 2023.

[1] Still, I would recommend to be somewhat elective in the jupyter notebook as well and to avoid cluttering it with repetitive examples.

[2] The same applies, maybe in even stronger form, to the code you are writing. Try to write it in such a way and add enough comments so that you are confident that you will still understand it in a few years.

Concerning language. If you are writing the report in English, make sure that your English is actually readable. An analogous statement applies, of course, if you are writing in Norwegian. Spell checkers are valuable tools, and it is a good idea to use one. However, do never rely solely on automated spell checking. If you find somebody else to look briefly over your report after you think it is finished, use this opportunity. It is easy to miss errors after one has worked on the same text over a long period of time.

Loops and repetitions. Writing is an iterative process. It might easily happen that you have to rewrite a paragraph or even a section, because it does not fit any more into the rest of the paper. Or, that it turns out that your paper exceeds the strict page limit you have for this project. Do not be afraid of this, as it is part of the writing process.

Rules of writing and good English. There are lots of different guides available on writing, and also on mathematical writing. I can recommend, for instance, the book by Higham [Hig98], or the excellent article by Halmos [Hal70]. Even if you do not find time to read these texts now, try to do so when/before writing your master thesis. However, only ever take all these guides as guidelines and break any rules outlined there, if you have a good reason for doing so.[3]

3. On writing mathematics

Mathematical writing has its peculiarities, which make mathematical texts somehow different from scientific texts in other disciplines.

Mathematical expressions. One of the most obvious differences between mathematical and non-mathematical texts is the frequent occurrence of mathematical phrases in the former, either within the running text like in the equation $x^2 + bx + c = 0$, or as a separate line like

$$x = -\frac{b}{2} \pm \sqrt{\frac{b^2}{4} - c}. \tag{1}$$

Here are just a few remarks and guidelines concerning their usage in mathematical texts:

- Mathematical symbols should always be used for a reason, usually either in order to make a sentence simpler to read—just try to imagine formulating the equation (1) in plain English—or because it would be difficult or impossible to obtain the same precision without mathematical symbols. However, if a statement can be easily expressed without symbols, there is no reason to introduce them by force. It is, for instance, perfectly fine to say "The square of every non-zero real number is positive." instead of "If $x \in \mathbb{R}$ and $x \neq 0$, then $x^2 > 0$."
- Avoid introducing notation if it is not necessary. For instance, in the sentence "Every complex matrix A has an eigenvalue", naming the matrix A serves no purpose whatsoever. It would be easier and shorter to simply write "Every complex matrix has an eigenvalue" instead.
- It is good practice, not to begin sentences with mathematical symbols, because they may make the text slightly more difficult to read. Similarly, always try to keep a minimal safety distance of at least one English word between separate mathematical expression. For instance, do avoid a sentence like "If $x > 0$, $x^2 > 0$," but rather write "If $x > 0$, then $x^2 > 0$."

[3] "I really want to" counts as a good reason, as long as you do not overuse it.

- If a mathematical derivation takes several steps, do not formulate it as a mere list of mathematical expressions, as you would do in a homework assignment or an exam.[4] Rather, comment briefly on what you are doing in between the formulas. This makes it much easier to follow the argumentation and understand what is happening.
- Mathematical expressions are part of the sentence structure, even if they are written in separate lines. This means in particular that punctuation marks like in (1) are necessary.
- Quantifiers like $\forall$ or $\exists$ and logical operators like $\Rightarrow$ or $\Longleftrightarrow$ should be used sparingly. Most of the time, it is better to replace them by the corresponding words.
- Be very careful with footnotes, as they can easily interfere with computations and for instance make equations like $a^2 + b^2 = c^{2^5}$ almost impossible to read correctly.

Obviously, together with its distant cousins *clearly*, *trivially*, and *it is easy to see*, is one of the most overused and misused words in mathematics. Often it is simply used as an abbreviation for "I do believe that what I write next is true. However, I am too lazy to explain why I think so, and I absolutely do not want to formulate an actual proof." Do not use the word in this meaning! Also, if you happen to write in your report that something is obvious, make absolutely sure that said something *is actually correct*.

We is not that bad. In mathematical writing, it is perfectly fine to use the word "we" both for referring to yourselves and to yourself or yourselves together with the reader, especially if other formulations using, for instance, the passive voice turn out to be much more cumbersome.[6]

4. Structure of numerical texts

A typical paper in numerical or applied mathematics has the following general structure:

- Abstract,
- Introduction,
- Main part,
- Numerical experiments,
- Conclusion,
- Bibliography.

I strongly advise to follow this tried and tested structure also in your project report.

Abstract. The abstract should give a very short summary of the main content and the main results of the paper. It should contain enough information to make a potential reader interested in the paper, and should also be seen as a short "advertisement" of the paper. Try to avoid mathematical expressions and also citations in the abstract, unless they are absolutely necessary.

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[4] In fact, I would, out of pure selfishness, extend the same advice to homework assignments and exams.

[5] This is not a mistake, but rather a very annoyingly placed reference to a footnote.

[6] Even if you are the sole author of a mathematical text, it is fine to use "we" as a pronoun. There is no need to convince F.D.C. Willard to coauthor the text with you.

Introduction. The introduction should describe the general setting of the paper and provide an overview of its main results. It is also very common to discuss the general structure of the paper. In scientific papers, and also in your master thesis, the introduction is also the place where you should compare your own results with related work in the same field; in the context of this project, however, this is not required.

In general, it is a good idea to write the introduction only after the rest of the paper is finished[7] and keep the introduction essentially independent. Also, avoid filling the introduction with technical definitions, but rather postpone as much mathematics as possible to the main part.

Main part. This is the main part of the paper, where "all the interesting mathematics happen." For this project, this will be the place where you define all the functions and the optimisation problem, discuss its properties, and also describe the numerical method used for its solution. This is *not* the place, where you would usually perform numerical experiments, though. Often, the main part will not just consist of a single section, but rather be split into several sections, each devoted to a particular subproblem. Use your common sense to find a reasonable structure.

Try to write short, but ensure that everything you say is correct. You will probably have to introduce some mathematical notation, but do so sparingly and try to keep your notation consistent (use similar symbols for similar objects).

Cite all your sources. Feel free to use all available resources, but never copy anything word by word and always include citations in your paper.[8]

Numerical experiments. This is the section, where you describe the numerical results you obtain by applying the methods described in the main part. Make sure that it is, in principle, possible to reproduce your experiments just from your descriptions in the paper. This means that you should provide all the details of the algorithm you have used, including all parameters, and also the settings in the specific experiments (configuration of the tensegrity structure, parameters of the structure and also the numerical algorithm, starting values of the iteration,...). Be careful that your numerical results do not contradict the results one obtains with your attached code. Be particularly careful with last-minute revisions of either your document or your code.

Feel free to use figures for the presentation of the results; in particular convergence rates often look far more convincing if presented with figures than with tables. If you do so, make sure that your figures are readable (both the plots themselves and the annotation).

Conclusion or summary. In this part, you should review once again the results obtained in the paper, and for instance discuss whether the numerical results were consistent with your theoretical predictions or not, and whether your method showed some "interesting" behaviour (e.g. failure in certain cases or, conversely, situations where it worked better than expected). In your master thesis, this would

[7]Or rather: after the first version of the rest of the paper is finished. You might need to revise the other sections after finishing the introduction.

[8]Similar to other fields, mathematics uses the convention that "common knowledge" need not be cited. There is, however, no general agreement as to what constitutes common knowledge. Pythagoras' theorem certainly is; an inverse function theorem for Gâteaux differentiable maps between graded Fréchet spaces probably is not, unless you are exclusively writing for specialists in this field.

For this project, you can assume that everything we have discussed in class is common knowledge and warrants no citation, and the same holds for the content of your basic mathematics and numerics classes. Also, you don't have to cite the project description in your report.

also be the place where you can discuss possible open (or new) questions and propose new directions of research.

Bibliography. Make sure that you cite all sources you have used during your project (don't cite the project description). As discussed above, it is not necessary to provide references for "common knowledge", that is, basic mathematical results that are well known to you and also to your intended audience.

I advice to use BibTeX for setting up the bibliography, as it works very well together with LaTeX and makes it easy to ensure a uniform style for the references. BibTeX entries for almost all mathematical articles and books published in the last hundred years can be found at `https://mathscinet.ams.org/mathscinet/` You will need an NTNU IP address in order to access this database, though.

References

[Hal70] P. R. Halmos. How to write mathematics. *Enseignement Math. (2)*, 16:123–152, 1970.

[Hig98] N. J. Higham. *Handbook of writing for the mathematical sciences*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, second edition, 1998.

[Kvæ16] A. Kværnø. Advice on how to write the report. Technical report, NTNU, Trondheim, 2016. `https://www.math.ntnu.no/emner/TMA4215/2016h/HowTo.pdf`.

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