

# Optimisation 1, Lecture 2

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Trondheim,  
January 13, 2023

## Previous lecture

Optimisation problem

$$\min_{x \in \Omega} f(x). \quad (P)$$

Global solution of  $(P)$ :

- A point  $x^* \in \Omega$  such that

$$f(x^*) \leq f(x) \quad \text{for all } x \in \Omega.$$

Local solution of  $(P)$ :

- A point  $x^* \in \Omega$  such that

$$f(x^*) \leq f(x) \quad \text{for all } x \in \Omega \text{ close to } x^*.$$

Strict global/local solution of  $(P)$ :

- Global/local solution where  $\leq$  can be replaced by  $<$ .

# Goals for today's lecture

- Existence of minimisers:
  - ▶ Coercivity.
  - ▶ Lower semi-continuity.
- Characterisation of local minima:
  - ▶ First order necessary conditions.
  - ▶ Second order necessary conditions.
  - ▶ Second order sufficient conditions.
  - ▶ (Error estimates.)
- Nagging about participation in the reference group.