## EXAM IN TMA4180 OPTIMISATION 1, 09$^{\text{TH}}$ MAY 2023

GRADING DOCUMENT

General remarks concerning the grading of the exam:

- The exam together with a detailed solution proposal can be found on the wiki page of the course.
- The weighting of the different (sub-)problems is provided in the exam.
- All answers have to be justified, and they should include enough details in order to see how they have been obtained.
  - If an answer comes with no or almost no explanation/justification, a significant number of points are deducted, even if the result happens to be correct.
- I usually do not subtract points for subsequent errors ("følgefeil"), unless they significantly simplify the rest of the problem. In particular, this applies to the following types of errors that were somewhat common:
  - Calculation errors in 1a leading to an incorrect argumentation as to why $f$ is non-convex: If several critical points were found in 1a, then it is correct to conclude that $f$ is non-convex; the same is true, if the Hessian at the critical point was found to be indefinite.
  - A wrong assessment of the function in problem 1 as convex (with always positive definite Hessian), leading to the conclusion that Newton's method will converge (quadratically).
- **I exercise discretion when awarding points for all the problems.**

Details for the different problems:

1a. **5 points** for finding the critical points (3 points for finding $(0, -1)$, 2 points for showing that this is the only critical point); **5 points** for discussing the properties (in order to get full marks, one needs to (correctly) show that $f$ is coercive; partial marks given for concluding something (correct) about the solution from the Hessian).

1b. **5 points** for explaining why the function is non-convex.

1c. **2 points** for applying the gradient descent step correctly; **4 points** for applying the conditions correctly; **4 points** for the calculations. Generally, I was stricter with deducting points for calculation errors, if they led to a solution that obviously contradicted the theory about line search methods (that is: if one did not obtain any upper or lower bound for the step length).

1d. **5 points** for discussing potential issues with Newton's method due to the non-convexity of $f$ (cannot guarantee descent directions, the Hessian might become singular); **5 points** for discussing the convergence rate (1 point for simply stating that Newton's method converges quadratically without considering the possibility of a singular Hessian at the solution); if the function was wrongly assessed as convex in part 1b, then one should conclude here that Newton's method will converge towards the unique solution unless the Hessian becomes singular.

2a. **2 points** for the sketch; **3 points** for LICQ.

*Date*: May 24, 2023.

2b. **4 points** for discussing the point $(3, 4)$; **2 points** for linearised feasible directions at $(5, 0)$; **4 points** for the tangent cone at $(5, 0)$.

2c. **3 points** for stating the KKT conditions correctly; **6 points** for finding the KKT points; **6 points** for discussing the properties of the KKT points (full marks require correct mathematical argumentation, e.g. second order conditions; partial marks given for heuristic/graphical arguments).

3. **3 points** for formulating the correct Lagrangian and stating the correct formula for the dual objective function; **5 points** for calculating the dual objective function; **2 points** for the dual problem, including the positivity constraint for the dual variable.
   Full marks are given, if the problem is *correctly* written as a linear programme (by splitting up the variable $x$ in its positive and negative part) and the dual of this linear programme is correctly computed; *at most* 5 points are given, if the problem is incorrectly rewritten as $\min c^T x$ such that $Ax \geq b$ with $c = (1, \ldots, 1)^T$ (and the remaining calculations are correct and well explained).

4a. **3 points** for existence; **2 points** for uniqueness. In the uniqueness, it is necessary to mention at some point that the feasible set is convex (e.g. by stating that the constraint is linear).

4b. **3 points** for showing coercivity; **2 point** for strict convexity.

4c. **1 point** for necessity of $Ax_{\lambda,\mu} = b$; **4 points** for sufficiency.
   There are different ways of showing sufficiency: One can for instance use the optimality conditions, or use the fact that the augmented Lagrangian coincides with $f$ on the whole feasible set. What is not enough, however, is to simply state that $\mathcal{L}_A(x_{\lambda,\mu}; \lambda, \mu) = f(x_{\lambda,\mu})$ if $Ax_{\lambda,\mu} = b$.

5. If the Pareto optimal points are found by means of the definition: **4 points** for the correct usage/statement of the definition; **6 points** for finding the Pareto-optimal solutions.
   If the Pareto optimal points are found by means of the weighted sum method: **5 points** for arguing why this method is applicable; **5 points** for finding the solutions.
   The Pareto optimal points can also be found by finding the image of $(f_1, f_2)$ and identifying the minimal points in that image. In this case, **4 points** are given for the correct image; **4 points** for the identification of the minimal points on the graph (in $\mathbb{R}^2$); **2 points** for the Pareto optimal points (in $\mathbb{R}$).

Grading scale:

A: 86–100 points.
B: 75–85 points.
C: 64–74 points.
D: 53–63 points.
E: 41–52 points.
F: 0–40 points.