Notation:

Want to minimise a function $f\colon \mathbb{R}^d \to \mathbb{R}$, possibly with equality constraints $c_i(x) = 0$, $i \in \mathcal{E}$, and inequality constraints $c_i(x) \geq 0$, $i \in \mathcal{I}$.

Lagrangian:

Lagrange parameters λ_i, $i \in \mathcal{E} \cup \mathcal{I}$,

$$\mathcal{L}(x, \lambda) = f(x) - \sum_{i \in \mathcal{E}} \lambda_i c_i(x) - \sum_{i \in \mathcal{I}} \lambda_i c_i(x)$$

Penalty and barrier functions:

Quadratic penalty function: $$Q(x; \mu) = f(x) + \frac{\mu}{2} \sum_{i \in \mathcal{E}} c_i(x)^2$$

Logarithmic barrier function: $$B(x; \beta) = f(x) - \beta \sum_{i \in \mathcal{I}} \ln\big(c_i(x)\big)$$

Augmented Lagrangian: $$\mathcal{L}_A(x, \lambda; \mu) = f(x) - \sum_{i \in \mathcal{E}} \lambda_i c_i(x) + \frac{\mu}{2} \sum_{i \in \mathcal{E}} \|c_i(x)\|_2^2$$

Line search methods:

Iteration $x_{k+1} = x_k + \alpha_k p_k$ with search direction $p_k \in \mathbb{R}^d$ and step length $\alpha_k > 0$.

Line search conditions:

Armijo: $$f(x_k + \alpha_k p_k) \leq f(x_k) + c_1 \alpha_k \langle \nabla f(x_k), p_k \rangle$$

Weak curvature: $$\langle \nabla f(x_k + \alpha_k p_k), p_k \rangle \geq c_2 \langle \nabla f(x_k), p_k \rangle$$

Strong curvature: $$\big|\langle \nabla f(x_k + \alpha_k p_k), p_k \rangle\big| \leq c_2 \big|\langle \nabla f(x_k), p_k \rangle\big|$$

Quasi-Newton updates:

Search direction $p_k = -H_k \nabla f(x_k)$ with:

$$s_k = x_{k+1} - x_k, \qquad y_k = \nabla f(x_{k+1}) - \nabla f(x_k).$$

DFP: $$H_{k+1} = H_k - \frac{(H_k y_k) \otimes (H_k y_k)}{\langle y_k, H_k y_k \rangle} + \frac{s_k \otimes s_k}{\langle y_k, s_k \rangle}$$

BFGS: $$H_{k+1} = (I - \rho_k s_k \otimes y_k) H_k (I - \rho_k y_k \otimes s_k) + \rho_k s_k \otimes s_k \quad \text{with } \rho_k = \frac{1}{\langle y_k, s_k \rangle}$$

SR1 : $$H_{k+1} = H_k + \frac{(s_k - H_k y_k) \otimes (s_k - H_k y_k)}{\langle s_k - H_k y_k, y_k \rangle}$$

Non-linear CG updates:

Search direction $p_{k+1} = -\nabla f(x_{k+1}) + \beta_{k+1} p_k$ with

Fletcher–Reeves: $$\beta_{k+1} = \frac{\|\nabla f(x_{k+1})\|_2^2}{\|\nabla f(x_k)\|_2^2}$$

Polak–Ribière: $$\beta_{k+1} = \frac{\langle \nabla f(x_{k+1}), \nabla f(x_{k+1}) - \nabla f(x_k) \rangle}{\|\nabla f(x_k)\|_2^2}$$

Hestenes–Stiefel: $$\beta_{k+1} = \frac{\langle \nabla f(x_{k+1}), \nabla f(x_{k+1}) - \nabla f(x_k) \rangle}{\langle \nabla f(x_{k+1}) - \nabla f(x_k), p_k \rangle}$$