Exercise #7

February 28, 2023

Problem 1.

Consider the constrained optimization problem

$$\min_{(x,y)} -x^2 - (y-1)^2 \qquad \text{such that} \qquad \begin{cases} y \geq Cx^2, \\ y \leq 2, \end{cases}$$

where $C > 0$ is some positive parameter.

a) Show that the point $(0,0)$ is a KKT point for all parameters $C > 0$ and that the LICQ is satisfied at $(0,0)$.

b) Formulate the second order necessary and sufficient optimality conditions for the point $(0,0)$. For which parameters C are these conditions satisfied? For which parameters C is the point $(0,0)$ a local minimum?

Problem 2.

Consider the constrained optimisation problem

$$\min_{(x,y)} \frac{1}{2}(x^2 + y^2) \qquad \text{subject to } xy = 1.$$

a) Find (by whatever means) the solutions of this problem. In addition, find the values of the corresponding Lagrange multipliers.

b) Formulate the unconstrained optimisation problem that results from the application of the quadratic penalty method with parameter $\mu > 0$. Solve these problems for all possible parameters μ and verify that the solutions converge to the solutions of the constrained optimization problem as $\mu \to \infty$.

c) Formulate the augmented Lagrangian for this constrained optimization problem and find (for all possible parameters $\lambda \in \mathbb{R}$ and $\mu > 0$) the global solutions of this (unconstrained) optimization problem. For which parameters does one recover the solution of the original constrained problem?

Problem 3.

Sketch the region $\Omega \subset \mathbb{R}^2$ defined by the inequalities

$$y \geq x^4 \qquad \text{and} \qquad y \leq x^3,$$

and compute the tangent cone and the set of linearized feasible directions for each point in Ω. For which points in Ω is the LICQ satisfied? (Note that this is same feasible region of the Problem 4.)

Problem 4.

Consider the constrained optimization problem

$$\min_{(x,y)} (x) \qquad \text{such that} \qquad \begin{cases} y \geq x^4, \\ y \leq x^3. \end{cases}$$

Find all KKT points and local minima for this optimization problem.

Problem 5.

Consider the constrained optimisation problem

$$\min_{(x,y)} (x + y) \text{ such that } x^2 + y^2 \leq 1.$$

Formulate a logarithmic barrier method for the solution of this constrained optimisation problem and compute its solution for each parameter $\mu > 0$ in the barrier functional.