

Exercise #10

March 21, 2023

Problem 1. (*Exercise 16.1 in N&W*)

Consider the quadratic programme

$$f(x, y) := 2x + 3y + 4x^2 + 2xy + y^2 \rightarrow \min$$

subject to

$$x - y \geq 0, \quad x + y \leq 4, \quad x \leq 3.$$

- Solve the quadratic program and sketch its geometry (that is, the domain of the problem and the level lines of the function f).
- What happens if one replaces the function f by $-f$? Does the problem still have solutions or local solutions?

Problem 2. (*Exercise 13.1 in N&W*)

Convert the following linear program to standard form:

$$\max_{x,y} (c^T x + d^T y) \text{ subject to } A_1 x = b_1, A_2 x + B_2 y \leq b_2, l \leq y \leq u,$$

where there are no explicit bounds on x .

Problem 3.

Assume that $f: \mathbb{R}^d \rightarrow \mathbb{R}$ is strictly concave and consider the optimization problem

$$\min_x f(x) \text{ subject to } Ax \geq b, \tag{1}$$

where $A \in \mathbb{R}^{m \times d}$ and $b \in \mathbb{R}^m$. Assume that x^* is a local solution of (1). Show that x^* is a vertex of the polyhedron $P = \{x \in \mathbb{R}^d : Ax \geq b\}$.