

Descent Methods

- goal: solve unconstrained optimization problem

$$\min_{x \in \mathbb{R}^n} f(x) \quad (\text{P})$$

where $f: \mathbb{R}^n \rightarrow \mathbb{R}$.

- approximation to a (local) minimizer: x^*
- iterative method
- idea: start with given point x_0

for $k = 0, 1, \dots$

Find x_{k+1} s.t. $f(x_{k+1}) < f(x_k)$

If some stopping criterion
is satisfied, then: STOP

end

Output: $x^* = x_{k+1}$.

$$x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \dots \rightarrow x_k \rightarrow x_{k+1}$$

$$x_{k+1} - x_k = p_k \iff x_{k+1} = x_k + p_k$$

↓
search direction

Definition: For a given $x_k \in \mathbb{R}^n$, a direction $p_k \in \mathbb{R}^n$ is called a descent direction

if there exists $\alpha > 0$ ($\alpha \in \mathbb{R}$) s.t.

$$f(x_k + \alpha p_k) < f(x_k) \quad \forall \alpha \in (0, \bar{\alpha}).$$

Interpretation: There is a small enough (but nonzero) amount that you can move in direction p_k and be guaranteed to decrease the function value.

Example: Gradient method: The direction p_k to move along at step k is chosen based on information from $\nabla f(x_k)$ (if f is differentiable).

Remark: When we speak of direction, the magnitude of the vector does not matter; e.g. $\nabla f(x)$, $3 \cdot \nabla f(x)$, $\nabla f(x) \cdot \frac{1}{\|\nabla f(x)\|}$ are all the same direction.