Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4180
Optimisation I
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Exercise set 6

1 Consider a constrained minimisation problem with continuously differentiable objective function and constraints. Which of the following statements are true?

(CQ : constraint qualification)

1. x^* is a global minimum $\implies$ x^* is a KKT point.
2. x^* is a local minimum and CQ holds $\implies$ x^* is a KKT point.
3. x^* is a KKT point and CQ holds $\implies$ x^* is a local minimum.
4. x^* is a global minimum and the problem is convex $\implies$ x^* is a KKT point.
5. x^* is a KKT point and the problem is convex $\implies$ x^* is a global minimum.

2 Assume that one wants to solve the optimisation problem

$$\max_x f(x) \qquad \text{such that} \qquad \begin{cases} c_i(x) = 0 & \text{for all } i \in \mathcal{E}, \\ c_i(x) \geq 0 & \text{for all } i \in \mathcal{I}. \end{cases}$$

How do the KKT conditions have to be modified such that one obtains (first order) necessary conditions for this maximisation problem?

3 Consider the constrained optimization problem

$$\min_{x,y} x^2 + y^2 \qquad \text{such that} \qquad \begin{cases} x + y \geq 1, \\ y \leq 2, \\ y^2 \geq x. \end{cases}$$

a) Formulate the KKT-conditions for this optimization problem.

b) Find all KKT points for this optimization problem.

c) Find all local and global minima for this optimization problem.

(Part b can be very tedious. One strategy is to consider all possible active sets and determine for each active set whether KKT-points exist. It can also be extremely helpful to sketch the feasible set and the function.)

4 (Problem 5, Exam 2014) Consider the following constrained optimization problem:

$$\min_{x,y} \frac{1}{2}(x^2 + y^2), \quad \text{s.t. } x - y - 1 = 0 \tag{1}$$

a) Find the globally optimal solution (x^*, y^*) for (1) (graphically, if you like). Also find the value of the Lagrange multiplier λ^* associated with the constraint at the globally optimal solution.

b) Formulate the unconstrained minimization problem corresponding to the application of the quadratic penalty method applied to (1). Solve the resulting unconstrained minimization problem for the penalty parameter $\mu = 2$.

c) State the augmented Lagrangian penalty function corresponding to (1) and the Lagrange multiplier λ and penalty parameter $\mu > 0$. Find the unconstrained global minimum of the augmented Lagrangian corresponding to $\lambda = 0.5$, $\mu = 2$. Compare the accuracy of the obtained approximate solutions to (1) with those obtained in the previous task.