TMA4180 Optimisation I

Geir Bogfjellmo

Department of Mathematics, Norwegian University of Science and Technology, Trondheim, Norway

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Course information

- https://wiki.math.ntnu.no/tma4180/2020v/start
- Grading: Final exam (70%) and project (30%)
- Literature: Nocedal & Wright *Numerical Optimisation* + lecture notes.
- Exercises: Not mandatory – but recommended.
- People:
  - Lecturer: Geir Bogfjellmo. Office 1250, SBII.
  - Teaching assistant: Esten Wøien.
  - Student assistant: Martin Ludvigsen.

Project

- Part of the grade is based on a project.
- Main focus of project:
  - Specific optimisation project.
  - Analysis of the problem
  - Implementation and analysis of optimisation algorithms.
- Format:
  - Two parts, counting 10% and 20%, respectively.
  - Groups of up to three students.
  - Timeframe TBA. (end of February and end of term).
  - Programming language: Python or Matlab recommended.

Practical Matters

- Exercises at **Nullrommet** (not R10).
- Excursion?
- Reference group.

Main topics

- Theory of optimisation:
  - Existence of minimisers.
  - Characterisation, necessary and sufficient conditions.
  - Convex functions and optimisation.
  - Optimality conditions for constrained optimisation.
- Methods for unconstrained optimisation:
  - Line search methods.
  - Trust region methods.
  - Gradient descent, Conjugate gradient, Newton and Quasi–Newton methods.
- Methods for constrained optimisation:
  - Penalty methods and Augmented Lagrangian.
  - Interior point methods.
  - Sequential quadratic Programming/simplex algorithm. (If time.)

Optimisation

Problem

Given an *objective function* $f : V \to \mathbb{R}$ and a *feasible region* $\Omega \subseteq V$, find $x^* \in \Omega$ such that $f(x^*)$ is minimal.

Possible difficulties:

- Local minima and stationary points.
- High dimensionality, memory issues.
- Accuracy/speed tradeoff.
- Inaccessibility of the function f.
- Ill-conditioned problems, stability.
- Constraints.

Continuous and discrete optimisation

Continuous optimisation:

- Ω (non-discrete) subset of $\mathbb{R}^d$.
- f mostly continuous and differentiable.
- Can use local properties (Taylor expansions).
- Cannot find minima exactly, only approximations.

Discrete optimisation:

- Ω finite subset of $\mathbb{Z}^d$.
- Finitely many $x \in \Omega$.
- No local properties.
- Complexity issues if Ω is large.
- Usually heuristic algorithms depending on problem.

This course: *only* continuous optimisation problems.

Free and constrained optimisation

Free optimisation:

- Ω is all of $\mathbb{R}^d$.
- Or constraints can be effectively ignored.
- Efficient and relatively simple algorithms possible.
- Algorithms for constrained optimisation are often based on reformulations as free optimisation problems.

Constrained optimisation:

- Omega defined by *constraints*.
- Constraints often given in the form of:
  - *Equality constraints* $c_i(x) = 0, i \in \mathcal{E}$.
  - *Inequality constraints* $c_i(x) \leq 0, i \in \mathcal{I}$.
- In general much harder than free optimisation.
- Non-smooth free optimisation problems can often be rewritten as smooth but constrained problems.