

Example problems for TMA4180, Optimisation I

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Problem 1

This is an example of a True/False problem. It will not be necessary or possible to provide justification. Consider the following optimisation problems. For each of the problems, determine if the problem is convex or not.

- a) $\min_{(x,y)} x^2 + y^2 + xy + y^4$
- b) $\min_{(x,y)} x + 2y$ s.t. $x^2 + \frac{y^2}{4} = 1$.
- c) $\min_{x \in \mathbb{R}^n} x^T A x - b^T x$ where A is an indefinite matrix.

Problem 2

This is an example of a Multiple Response problem. It will not be necessary or possible to provide justification for your answer. Consider the constrained optimisation problem

$$\min_{(x,y)} -y^2 - x^2 + 2x \quad \text{s.t.} \quad \begin{cases} 2 - x^2 - y^2 \geq 0 \\ x + y \geq 0 \end{cases}$$

Determine which of the following points are KKT points:

- $(0, 0)$
- $(0, 1)$
- $(1, -1)$
- $(-1, 1)$

- $(\sqrt{2}, 0)$
- $(0, \sqrt{2})$
- $(-\sqrt{2}, 0)$

Problem 3

This problem is an essay problem. You can enter mathematical formulae in your answer, but it is cumbersome.

A discretisation of a partial differential equation called the p -Laplacian equation leads to an optimisation problem

$$\min_{x \in \mathbb{R}^N} \sum_{i=1}^N \sum_{j=1}^N \frac{\alpha_{i,j}}{2} |x_i - x_j|^p \quad \text{subject to} \quad x_i = g_i \text{ for } i \in \mathcal{B}, \quad (1)$$

where p is a real number, $p > 1$, $\alpha_{i,j} = \alpha_{j,i}$ are nonnegative real numbers, and $\mathcal{B} \subset \{1, 2, \dots, N\}$. Typically $|\mathcal{B}| \ll N$.

Which method would you use to solve (1) numerically for a moderate value of N , say $N = 400$? Does it depend on p ? Justify your answers.

Problem 4

This is another example of a True/False question Determine whether the following statement is True or False.

If $f: \mathbb{R}^n \rightarrow \mathbb{R}$ is C^1 and x^* is such that $\nabla f(x^*) = 0$, then x^* is a local minimum of x^* .

File upload answer problem

There will be a part where you will upload a pdf document (handwritten on paper and photographed, handwritten on pad...) with your answer. See older exams for examples of problems. Exams 2014–2018 are most relevant.