Norwegian University of Science and Technology
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TMA4180 Optimization I Spring 2018

Exercise set 8

1 For the following two examples, sketch the region Ω defined by the constraints and compute for each point in Ω both the tangent cone and the set of linearized feasible directions. For which points is the LICQ satisfied?

a) The region $\Omega \subset \mathbb{R}^2$ defined by the inequalities

$$y \geq x \qquad \text{and} \qquad y^4 \leq x^3.$$

b) The region $\Omega \subset \mathbb{R}^2$ defined by the inequalities

$$y \geq x^4 \qquad \text{and} \qquad y \leq x^3.$$

2 Determine the cone of feasible directions (radial cone) $R_\Omega(0)$, the tangent cone $T_\Omega(0)$ and the cone of linearized feasible directions $F_\Omega(0)$ for the following sets Ω. Determine, which constraint qualifications hold at $0 \in \Omega$.

a) $\Omega = \{\, x \in \mathbb{R}^2 \mid x_1 \geq 0, -(x_1 - 1)^2 - x_2^2 + 1 \geq 0 \,\}$.

b) $\Omega = \{\, x \in \mathbb{R}^2 \mid x_1 \geq 0, x_2 \geq 0, -x_1 x_2 \geq 0 \,\}$.

c) $\Omega = \{\, x \in \mathbb{R}^2 \mid x_1^3 - x_2 \geq 0, x_2 - x_1^5 \geq 0, x_2 \geq 0 \,\}$.

d) $\Omega = \{\, x \in \mathbb{R}^2 \mid x_2 \geq 0, (x_1 - 1)^2 + x_2^2 - 1 = 0 \,\}$.

Now consider the function $f(x) = x_1$.

e) Find the points of global minimum for f over Ω given in examples a)–d). Verify that the optimality conditions $\forall p \in T_\Omega(0) : \nabla f(0)^\top p \geq 0$ are satisfied.