Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4180 Optimisation I Spring 2018

Exercise set 6

1 In this exercise, we study the Gauß–Newton method for solving the least-squares problem corresponding to the (overdetermined and inconsistent) system of equations

$$\begin{aligned} x + y &= 1, \\ x - y &= 0, \\ xy &= 2. \end{aligned}$$

To that end, we define

$$\begin{aligned} r_1(x,y) &:= x + y - 1, \\ r_2(x,y) &:= x - y, \\ r_3(x,y) &:= xy - 2, \end{aligned}$$

and

$$f(x,y) := \frac{1}{2}\sum_{j=1}^{3} r_j(x,y)^2.$$

We denote moreover by $J = J(x,y)$ the Jacobian of $r = (r_1, r_2, r_3)\colon \mathbb{R}^2 \to \mathbb{R}^3$.

a) Show that the function f is non-convex, but that it has a unique minimiser (x^*, y^*).

b) Show that the matrix $J^\top J$ required in the Gauß–Newton method is positive definite for all x, y.

c) Show that the Gauß–Newton method with Wolfe line search for the minimisation of f converges for all initial values (x_0, y_0) to the unique solution of the non-linear least squares problems.

d) Perform one step of the Gauß–Newton method (without line search) for the solution of this least-squares problem. Use the initial value $(x_0, y_0) = (0, 0)$.

2 Let

$$f(x) = x_1^4 + 2x_2^4 + x_1x_2 + x_1 - x_2 + 2.$$

Starting at the point $x_0 = (0,0)$ compute explicitly one step for the trust region method with the model function $m(p) = f(x_0) + g^\top p + \frac{1}{2}p^\top Bp$, where $g = \nabla f(x_0)$, $B = \nabla^2 f(x_0)$, and the trust region radius $\Delta = 1$.

3 Let

$$f(x) = \tfrac{1}{2}x_1^2 + x_2^2,$$

put $x_0 = (1,1)$, and define the model function $m(p) = f(x_0) + g^\top p + \frac{1}{2}p^\top Bp$ with $g = \nabla f(x_0)$ and $B = \nabla^2 f(x_0)$.

a) Compute explicitly the next step p in the trust region method using values of $\Delta = 2$ and $\Delta = 5/6$.

b) Compute for all $\Delta > 0$ the next step in the dogleg method.