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TMA4180 Optimization I Spring 2016

Exercise set 5

The first three of the following exercises are concerned with the trust region method. You find details of the method (and also some more exercises) in Nocedal & Wright, Chapter 4. The last exercise is concerned with constrained optimisation, or rather with the concepts of tangent cones and linearized feasible directions. In particular, this exercise intends to show the problems that can occur, if the LICQ fails.

1 Let

$$f(x) = \frac{1}{2}x_1^2 + x_2^2,$$

and let $x_0 = (1, 1)$, $g = \nabla f(x_0)$, $B = \nabla^2 f(x_0)$.

a) Compute explicitly the next step in the trust region method using values of $\Delta = 2$ and $\Delta = 5/6$.

b) Compute for all $\Delta > 0$ the next step in the dogleg method.

2 Denote, for $\Delta > 0$, by $p^*(\Delta)$ the solution of the minimization problem

$$\min_{\|p\| \leq \Delta} m(p),$$

where

$$m(p) = f(x_0) + \nabla f(x_0)^T p + \frac{1}{2} p^T \nabla^2 f(x_0) p$$

for some function $f \colon \mathbb{R}^n \to \mathbb{R}$ and $x_0 \in \mathbb{R}^n$. Show that, in general, the curve $\Delta \mapsto p^*(\Delta)$ is not planar.

(Hint: Choose for instance $f(x) = x_1^2 + 2x_2^2 + 3x_3^2$.)

(Remark: As a consequence of this exercise, we see that the two-dimensional subspace minimization will, in general, not produce the optimal steps.)

3 Exercise 4.7 in Nocedal & Wright.

$\boxed{4}$ For the following two examples, sketch the region Ω defined by the constraints and compute for each point in Ω both the tangent cone and the set of linearized fesible directions. For which points is the LICQ satisfied?

a) The region $\Omega \subset \mathbb{R}^2$ defined by the inequalities

$$y \geq x \qquad \text{and} \qquad y^4 \leq x^3.$$

b) The region $\Omega \subset \mathbb{R}^2$ defined by the inequalities

$$y \geq x^4 \qquad \text{and} \qquad y \leq x^3.$$