TMA4180
Optimization I
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Exercise set 2

1 Consider the quadratic function

$$f(x) = \frac{1}{2}x^T Q x - b^T x$$

with $Q \in \mathbb{R}^{n \times n}$ symmetric and positive definite.

a) Compute the gradient and the Hessian of the function f, verify that f is strictly convex, and find the unique global minimum of f.

b) Verify that Newton's method (without line search) finds the minimum of f after one step.

c) Assume that f is minimized using Newton's method with backtracking line search. Show that in this case unit steps are accepted by the method if and only if $c_1 < 1/2$.

2 Consider the function

$$f(x, y) = 2x^2 + y^2 - 2xy + 2x^3 + x^4.$$

a) Compute all stationary points of f and find all global or local minimizers of f.

b) Consider the gradient descent method with backtracking for the minimization of f. Use the parameters $\rho = 1/2$ and $c_1 = 1/4$. Compute one step with starting value $(x_0, y_0) = (-1, 0)$. Does the method converge to a minimizer of f?

c) Consider Newton's method with backtracking for the minimization of f. Use the parameters $\rho = 1/2$ and $c_1 = 1/4$. Compute one step with starting value $(x_0, y_0) = (-1, 0)$. Does the method converge to a minimizer of f?

3 Implement both a gradient descent method and Newton's method with backtracking for the minimization of the Rosenbrock function

$$f(x, y) := 100(y - x^2)^2 + (1 - x)^2.$$

Whenever the Newton direction is not a descent direction, use the steepest descent direction instead.[1]

(You may use any programming/script language you want, but at your own risk. During the exercise sessions, help will be provided only for MATLAB.*)*

[1] For making line search methods work, it is necessary to guarantee that the diection p_k is in each step a descent direction. However, it turns out that the method suggested in this exercise actually may lead to a non-convergent iteration. Better (but much more complicated) methods can be found in Nocedal & Wright, Section 3.4).