Norwegian University of Science and Technology
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TMA4180 Optimization Theory Spring 2015

Exercise set 7

$\boxed{1}$ (See Nocedal & Wright, Exercise 12.17) Assume that the KKT conditions are satisfied at a point x^* where the LICQ holds. Show that the corresponding Lagrange parameter λ^* is unique. In addition, show that the Lagrange parameters need not be unique at KKT points where the LICQ fails.

$\boxed{2}$ (See Nocedal & Wright, Exercise 12.20) Find (using the first and second order optimality conditions) all local and global solutions of the optimization problem

$$xy \to \min \qquad \text{such that} \qquad x^2 + y^2 = 1.$$

$\boxed{3}$ Consider the constrained optimization problem

$$-x^2 - (y-1)^2 \to \min \qquad \text{such that} \qquad \begin{cases} y \geq Cx^2, \\ y \leq 2, \end{cases}$$

where $C > 0$ is some positive parameter.

a) Show that the point $(0,0)$ is a KKT point for all parameters $C > 0$ and that the LICQ is satisfied at $(0,0)$.

b) Formulate the second order necessary and sufficient optimality conditions for the point $(0,0)$. For which parameters C are these conditions satisfied? For which parameters C is the point $(0,0)$ a local minimum?

$\boxed{4}$ We consider the constrained optimization problem

$$xy \to \min \qquad \text{such that} \qquad \begin{cases} y \geq x, \\ y^4 \leq x^3, \end{cases}$$

from problem 3 of the exercise set 6. We have already shown that the point $(0,0)$ is a KKT point (with some Lagrange parameters λ^*) and the unique (local and global) solution of this optimization problem, but the LICQ fails at this point.

a) Compute the critical cone at $(0,0)$ as defined in the lecture and Nocedal & Wright, and show that there exist directions d contained in the critical cone for which $d^T \nabla^2 \mathcal{L}((0,0); \lambda^*) d < 0$.

b) Show that $d^T \nabla^2 \mathcal{L}((0,0); \lambda^*) d \geq 0$ for all vectors d contained in the tangent cone to the feasible set at $(0,0)$.