Norwegian University of Science and Technology
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TMA4180 Optimization Theory Spring 2015

Exercise set 6

The following exercises are all concerned with constrained optimization and the first order optimality conditions (that is, KKT conditions). Exercise 1 asks you to reformulate a non-smooth optimization problem as smooth optimization problem with constraints. Exercises 2–4 are concerned with the actual solution of (relatively simple) constrained optimization problems. Here, Exercise 2 is the “typical case” that you can expect to appear in most practical applications, whereas Exercises 3 and 4 deal with slightly problematic situations.

1 An alternative to standard linear regression is ℓ^1*-regression* or *robust regression*. Here one assumes that one is given data points (t_i, s_i), $1 \le i \le m$, which one wants to approximate by an affine function $g(t) := at + b$. In contrast to the standard method, however, one defines the coefficients a and b as solutions of the optimization problem

$$f(a,b) := \sum_{i=1}^{m} |at_i + b - s_i| \to \min .$$

a) Reformulate this non-smooth, unconstrained optimization problem as a linear programme (i.e., linear optimization problem with linear constraints).

b) Formulate the KKT-conditions for the constrained problem.

2 Consider the constrained optimization problem

$$x^2 + y^2 \to \min \qquad \text{such that} \qquad \begin{cases} x + y \ge 1, \\ y \le 2, \\ y^2 \ge x. \end{cases}$$

a) Formulate the KKT-conditions for this optimization problem.

b) Find all KKT points for this optimization problem.

c) Find all local/global maxima and minima for this optimization problem.

(Part b can be very tedious. One strategy is to consider all possible active sets and determine for each active set whether KKT-points exist. It can also be extremely helpful to sketch the feasible set and the function. Note also that we have briefly discussed this particular example in the first lecture.)

3 Consider the constrained optimization problem

$$xy \to \min \qquad \text{such that} \qquad \begin{cases} y \geq x, \\ y^4 \leq x^3. \end{cases}$$

a) Sketch the region Ω defined by the constraints and compute for each point in Ω both the tangent cone and the set of linearized feasible directions. For which points is the LICQ satisfied?

b) Find all KKT points, local minima, and local maxima for this optimization problem.

4 Consider the constrained optimization problem

$$x \to \min \qquad \text{such that} \qquad \begin{cases} y \geq x^4, \\ y \leq x^3. \end{cases}$$

a) Sketch the region Ω defined by the constraints and compute for each point in Ω both the tangent cone and the set of linearized feasible directions. For which points is the LICQ satisfied?

b) Find all KKT points, local minima, and local maxima for this optimization problem.