Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4180 Optimization Theory Spring 2015

Exercise set 3

All of the following exercises are concerned with the trust region method. You find details of the method (and also some more exercises) in Nocedal & Wright, Chapter 4.

1 Let

$$f(x) = \frac{1}{2}x_1^2 + x_2^2,$$

and let $x_0 = (1, 1)$, $g = \nabla f(x_0)$, $B = \nabla^2 f(x_0)$.

a) Compute explicitly the next step in the trust region method using values of $\Delta = 2$ and $\Delta = 5/6$.

b) Compute for all $\Delta > 0$ the next step in the dogleg method.

2 Implement the dogleg method for the minimization of the Rosenbrock function. Choose B_k to be the Hessian of f in each step.

3 Denote, for $\Delta > 0$, by $p^*(\Delta)$ the solution of the minimization problem

$$\min_{\|p\| \leq \Delta} m(p),$$

where

$$m(p) = f(x_0) + \nabla f(x_0)^T p + \frac{1}{2} p^T \nabla^2 f(x_0) p$$

for some function $f\colon \mathbb{R}^n \to \mathbb{R}$ and $x_0 \in \mathbb{R}^n$. Show that, in general, the curve $\Delta \mapsto p^*(\Delta)$ is not planar.

(Hint: Choose for instance $f(x) = x_1^2 + 2x_2^2 + 3x_3^2$.)

(Remark: As a consequence of this exercise, we see that the two-dimensional subspace minimization will, in general, not produce the optimal steps.)

4 Exercise 4.5 in Nocedal & Wright.

5 Exercise 4.7 in Nocedal & Wright.