Norwegian University of Science
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TMA4180
Optimization Theory
Spring 2015

Exercise set 1

1 *(See N&W, Exercise 2.1).* The *Rosenbrock function* is defined as

$$f(x) := 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$

a) Compute the gradient and the Hessian of the Rosenbrock function.

b) Show that the point $(1,1)$ is the unique (global and local) minimizer of f.

2 Let I be any index set and let $f_i \colon \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ be lower semi-continuous. Show that the function $f \colon \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ given as

$$f(x) = \sup_{i \in I} f_i(x)$$

is lower semi-continuous.

3 For the following functions, decide whether they are lower semi-continuous or coercive, and whether they attain a global minimizer:

a) The function $f \colon \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = x^4 - 20x^3 + \sup_{k \in \mathbb{N}} \sin(kx).$$

b) The function $g \colon \mathbb{R} \to \mathbb{R}$ given by

$$g(x) = e^x - \frac{1}{x^2 + 1}.$$

c) The function $h \colon \mathbb{R}^2 \to \mathbb{R}$ given by

$$h(x) = x_1^2(1 + x_2^3) + x_1^2.$$

4 *(See N&W, Exercise 2.8)* Assume that $f \colon \mathbb{R}^n \to \mathbb{R}$ is a convex function. Show that the set of minimizers of f is convex.

5 Show that a strictly convex function $f \colon \mathbb{R}^n \to \mathbb{R}$ has at most one global minimizer. In addition, find a strictly convex function that has no global minimizer at all.

6 Assume that $f\colon \mathbb{R}^n \to \mathbb{R}$ is convex, continuous, and bounded below. Show that for every $\lambda > 0$ the function

$$f_\lambda(x) := f(x) + \lambda \|x\|^2$$

has a unique global minimizer.

(*Additional challenge: Is it possible to drop the condition that f is bounded below?*)

7 Show that the requirement that $0 < c_1 < c_2 < 1$ in the definition of the Wolfe conditions is essential.

More precisely: Show that it is possible that no positive step length satisfies the Wolfe conditions if either $c_2 \geq 1$ or $c_1 > c_2$.