1. Project 1. Math 4175, Spring semester 2021

Due Monday February 29, 2021. Email: Berit Stensones, berit.stensones@ntnu.no

Problem 1. *Find* $\frac{1}{2\pi i}\int_{|z|=4}\frac{\cos(\pi z/2)}{z(z-1)^2}dz$

Problem 2. *Find a conformal map from* $D = \{z; 0 < |z| < 1\}$ *and* $0 < \arg(z) < \pi/4\}$ *to the upper half plane* $\{z; Im\ z > 0\}$.

Problem 3. *Prove that if* f *is analytic in a domain* Ω *and there exists a point* $a \in \Omega$ *such that* $f^{(n)}(a) = 0$ *for all* $n = 0, 1, 2, \ldots$ *then* $f(z) = 0$ *for all* $z \in \Omega$.

Problem 4. *(a) Let* $\alpha \in \mathbb{C}, |\alpha| < 1$, *show that the map*

$$\phi_\alpha(z) = \frac{z-\alpha}{1-\overline{\alpha}z}$$

sends the unit circle $\{|z| = 1\}$ *to the unit circle and the unit disc* $\{|z| < 1\}$ *to the unit disc and* α *to* 0.

(b) Use (a) together with the Schwarz Lemma to show that if $f : \Delta(0,1) \to \Delta(0,1)$ *is analytic and there are* $\alpha, \beta \in \Delta(0,1), \alpha \neq \beta$ *such that* $f(\alpha) = \alpha, f(\beta) = \beta$ *then* $f(z) = z$ *for all* $z \in \Delta(0,1)$. *(Hint: Do this first when* $\alpha = 0$.*)*

Problem 5. *Let* $P(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0$. *Show that* P *has* n *roots in* $\mathbb{C}$ *counting multiplicities. (Hint: Compare* P *to* z^n *on a large circle centered at* 0.*)*