

PROJECT 2. MATH 4175, SPRING SEMESTER 2021

Due Monday April 19, 2021. Send solutions to berit.stensones@ntnu.no

Problem 1. Find a conformal map from $D = \{z; |z| < 1 \text{ and } z \notin (-1, 0]\} = \Delta(0, 1) \setminus (-1, 0]$ to the first quadrant.

Problem 2. Evaluate the following integrals and show the estimates needed.

a)

$$\int_0^{2\pi} \frac{dx}{2 + \cos^2 x}$$

b)

$$\int_{-\infty}^{\infty} \frac{x^4 + 1}{x^6 - 4x^3 + 6}$$

Problem 3. Assume that f and g are analytic in a neighborhood of $\overline{\Delta} = \{z; |z| \leq 1\}$. Show that if $e^{f(z)} = e^{g(z)}$ on $\partial\Delta$ (the boundary of $\overline{\Delta}$), then $f(z) = g(z) + i2\pi k$ for some integer k and for all $z \in \overline{\Delta}$.

Problem 4. Prove that $\sum_{n=1}^{\infty} nz^{3n-1}$ converges uniformly to $\frac{z^2}{(1-z^3)^2}$ on every compact set in the unit disc.

Problem 5. Show that $u(x, y) = x^5 - 10x^3y^2 + 5xy^4 + x^2 - y^2$ is harmonic and find

$$\int_0^{2\pi} u(1 + \cos t, \sin t) dt$$