Example 1:

Map $\{z : 0 < \operatorname{Im} z < \frac{\pi}{2}\} = D$

Onto the unit disc

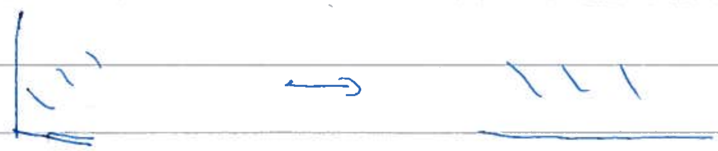
This can be done several ways

Let $z \xrightarrow{\varphi_1} e^z$, then

$$\varphi_1(D) = \{w : \operatorname{Re} w > 0 \text{ and } \operatorname{Im} w > 0\}$$

$$w = \varphi_1(z)$$

Next send $w \xrightarrow{\varphi_2} w^2$



Then $\varphi_2 \circ \varphi_1(D) = \{\xi : \operatorname{Im} \xi > 0\}$

Finally $\xi \xrightarrow{\varphi_3} \dfrac{\xi - i}{\xi + i}$ then

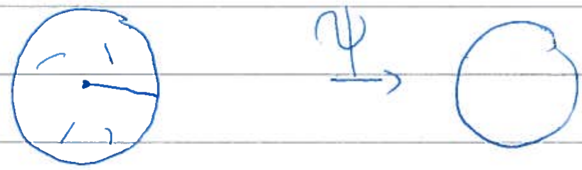
$$\varphi_3 \circ \varphi_2 \circ \varphi_1(D) = \{\eta : |\eta| < 1\}$$

$$\varphi_3 \circ \varphi_2 \circ \varphi_1(z) = \varphi_3 \circ \varphi_2(e^z) = \varphi_3(e^{2z})$$

$$= \frac{e^{2z} - i}{e^{2z} + i}$$

Example 2

Map $D = \{|z|<1\} \setminus \{z = x : 0 \le x < 1\}$
to the unit disc

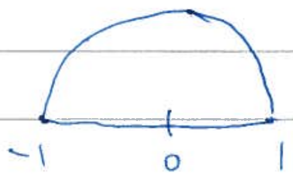


D can be described as

$\{z : 0 < |z| < 1$ and $0 < \arg z < 2\pi\}$

First step send $z \xrightarrow{\varphi_1} \sqrt{z}$

Then $\varphi_1(D) = \{z : |z|<1$ and $\operatorname{Im} z > 0\}$



$w = \varphi_1(D)$

Next we use $T(w)$ such that

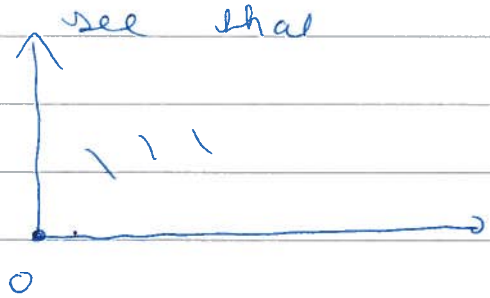
$-1 \to 0$, $0 \to 1$ and $1 \to \infty$

then the line segment $(-1, 1)$
is mapped to the positive
real axis

Now the upper half circle will be mapped to a ray starting at 0 ending up at ∞.

The angle between the half circle and the line segment is $\frac{\pi}{2}$ so we see that

$T(\text{half disc})$



must be the region $\{\xi : \mathrm{Re}\,\xi > 0 \text{ and } \mathrm{Im}\,\xi > 0\}$

$$\xi = T(w) = \frac{w+1}{1-w}$$

Next send $\xi \xrightarrow{\varphi_3} \xi^2$ then

$\varphi_3 \circ T \circ \varphi_1 = \{\eta : \mathrm{Im}\,\eta > 0\}$ so

we can use $\varphi_4(\eta) = \dfrac{\eta - i}{\eta + i}$.

Let $\psi(z) = \varphi_4 \circ \varphi_3 \circ T \circ \varphi_1(z)$