NTNU

Norwegian University of
Science and Technology

Department of Mathematical Sciences

Examination paper for **TMA4175 Complex Analysis**

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Examination date: . August 2019

Examination time (from–to): 09:00–13:00

Permitted examination support material: C: One yellow A4-sized sheet of paper stamped by the Department of Mathematical Sciences. On this sheet the student may write whatever he wants. Specific basic calculator allowed. No other aids permitted.

Other information:
The seven problems 1, 2, 3a, 3b, 4, 5, and 6 have equal weight.

Language: English

Number of pages: 1

Number of pages enclosed: 0

Checked by:

Informasjon om trykking av eksamensoppgave	
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Date Signature

Problem 1 Evaluate the integral

$$\int_{-\infty}^{\infty} \cos(x^2)\, dx.$$

Hint: $\int_0^\infty e^{-x^2}\, dx = \sqrt{\pi}/2$.

Problem 2 Map the domain $0 < \phi < \frac{\pi}{3}$ in the z-plane ($z = re^{i\phi}$) conformally onto the unit disk $|w| < 1$.

Problem 3 Consider the series

$$S(z) = \sum_{n=0}^{\infty} (2 + (-1)^n)^n\, z^n.$$

a) Determine the radius R of convergence.

b) Find a function $f(z)$ that is meromorphic in the whole plane and $f(z) = S(z)$ when $|z| < R$.

Problem 4 Find all values of

$$i^i.$$

Problem 5 Show that

$$\zeta(z) = \frac{1}{\Gamma(z)} \int_0^\infty \frac{t^{z-1}\, dt}{e^t - 1}, \qquad z = x + iy,$$

holds when $x > 1$. Here $\zeta(z) = \sum n^{-z}$.

Problem 6 Assume that the analytic function $f(z)$ satisfies

$$|f(z)| \le 2019 \exp(100|z|) \qquad \text{for all} \quad z$$

and that $f(m + in) = 0$ for all integers m and n. Show that $f(z)$ is constant.