NTNU
Norwegian University of Science and Technology

Department of Mathematical Sciences

Examination paper for **TMA4175 Complex Analysis**

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Examination date: 29 May 2019

Examination time (from–to): 9.00 -13.00

Permitted examination support material: C: One yellow A4-sized sheet of paper stamped by the Department of Mathematical Sciences. On this sheet the student may write whatever he wants. Specific basic calculator allowed. No other aids permitted.

Other information:
There are 6 problems of equal weight

Language: English

Number of pages: 2

Number of pages enclosed: 0

Checked by:

Date Signature

Problem 1 Does the double series

$$\sum_{m,n=-\infty}^{\infty} \frac{1}{(m+ni)^3} \qquad (i^2=-1)$$

converge? The term with $m=0, n=0$ is excluded. (Proof required.)

Problem 2 Suppose that the function $f(z)$ is analytic (= holomorphic) in the whole complex plane and that it has the periods 1 and i, i.e.,

$$f(z+1) \equiv f(z), \quad f(z+i) \equiv f(z)$$

for all complex numbers z. Explain why the function must be a constant.

Problem 3 Map the domain bounded by the two circles

$$|z| = 2 \quad \text{and} \quad \left|z - \frac{1}{2}\right| = \frac{1}{2}$$

conformally onto the ring domain

$$R < |w| < 1.$$

Determine the radius R of the inner circle in the w-plane.

Problem 4 Calculate the integral

$$\frac{1}{2\pi i}\int_{1-i\infty}^{1+i\infty} \frac{2^z}{z(z+1)}\,dz$$

over the vertical line $\mathfrak{Re}(z) = 1$. *Hint:* Use a suitable half-circle as a contour.

Problem 5 Show that

$$1 + 2^{-s} + 3^{-s} + \cdots = \frac{1}{\Gamma(s)}\int_0^\infty \frac{x^{s-1}}{e^x - 1}\,dx,$$

when $s > 1$. (Recall that

$$\Gamma(s) = \int_0^\infty e^{-t}t^{s-1}\,dt, \quad s > 1.)$$

Problem 6 Write $\cos(z)$ as an infinite product

$$\cos(z) \;=\; e^{g(z))} z^m \prod_n \left(1 - \frac{z}{a_n}\right) e^{\frac{z}{a_n}}$$

according to Hadamard's Theorem. Why does the product converge? Determine the a_n, the integer m, and construct the function $g(z)$.

Good luck!