[Christoffel – Schwarz]

The mapping must be of the form

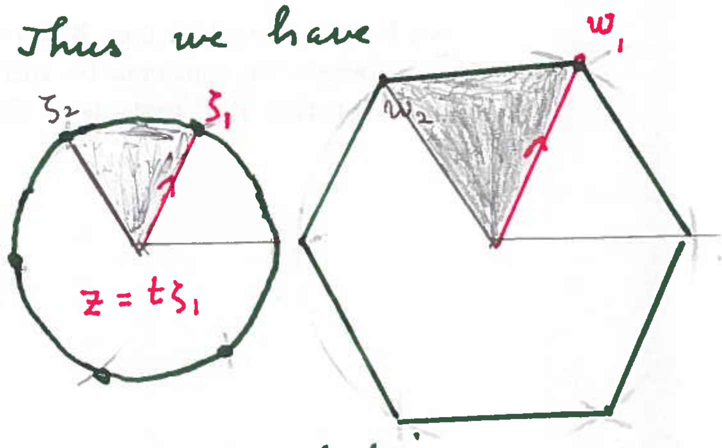
$$A\int (z-\zeta_1)^{-2/n}(z-\zeta_2)^{-2/n}\cdots(z-\zeta_n)^{-2/n}\,dz .$$

Now

$$z^n-1 = (z-1)(z-e^{\frac{2i\pi}{n}})(z-e^{\frac{2i\pi}{n}\cdot 2})\cdots(z-e^{\frac{2i\pi}{n}(n-1)})$$

with the roots of unity. Thus we have

$$A\int (z^n-1)^{-2/n}\,dz .$$



We get

$$w = \int_0^z \frac{d\zeta}{(1-\zeta^n)^{2/n}}$$

upon renaming the variable and taking

$$A\,(-1)^{-2/n} = 1 .$$

The point $\zeta_k = e^{\frac{2\pi i}{n}k}$ is mapped to the corner

$$w_k = e^{2\pi i k/n}\int_0^1 \frac{dt}{(1-t^n)^{2/n}} \qquad (\zeta = e^{\frac{2\pi i k}{n}}t)$$

The side length is

$$|w_{k+1}-w_k| = \left|2i\,e^{\frac{2\pi i k}{n}}e^{\frac{\pi i}{n}}\sin\left(\frac{\pi}{n}\right)\right|\int_0^1\frac{dt}{(1-t^n)^{2/n}}$$

$$= 2\sin\left(\frac{\pi}{n}\right)\int_0^1\frac{dt}{(1-t^n)^{2/n}}$$