NTNU

Norwegian University of
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Department of Mathematical Sciences

Examination paper for **TMA4175 Complex Analysis**

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Examination date: 24 May 2018

Examination time (from–to): 9.00 -13.00

Permitted examination support material: B:Alle trykte og haandskrevne hjelpemidler tillatt. Bestemt enkel kalkulator tillatt.

Other information:
There are 7 problems of equal weight

Language: English

Number of pages: 1

Number of pages enclosed: 0

Checked by:

Date Signature

Problem 1 Determine the constants a and b so that the polynomial

$$ax^3 + 3bx^2y + 3xy^2 + 2y^3$$

is the real part of an analytic function. Construct the function.

Problem 2 Map the first quadrant $\mathfrak{Re}\, z > 0, \mathfrak{Im}\, z > 0$ conformally onto the unit disk.

Problem 3 Calculate the integral

$$\int_0^\infty \frac{x^{\frac{1}{3}}}{(4+x)^2}\, dx.$$

Problem 4 Asssume that

$$f(z) = a_0 + a_1 z + a_2 z^2 + \cdots \quad \text{and} \quad |f(z)| < \frac{1}{1-|z|}$$

when $|z| < 1$. Show that

$$|a_n| < (n+1)\Big(1 + \frac{1}{n}\Big)^n, \qquad n = 1, 2, 3, ...$$

Problem 5 Find all analytic functions $f(z)$ satisfying

$$|f(z)| \le 2018\, |\sin(z)| \quad \text{when} \quad z \in \mathbb{C}.$$

Problem 6 The function $g(z)$ is analytic and $|g(z)| < 1$ when $|z| < 1$. Furthermore, $g(0) = 0$. Does the series

$$g(z) + g(z^2) + g(z^3) + \cdots$$

converge when $|z| < 1$?

Problem 7 Determine a function $h(z)$ so that the infinite product

$$\prod_{k=1}^{\infty} \Big(1 + \frac{z}{k}\Big)^k \exp\Big(\frac{z^2}{2k} + h(z)\Big)$$

converges.

Good luck!